Non-Circular Pressure Vessels

— Some guidance notes for designers

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Introduction

The aim of this article is to set down, in readily available fashion, the fundamental theory needed for the design of some typical pressure vessels of non-circular cross-section.

Of these the most common are the rectangular section tanks. They are often used as bulk storage containers or as baths in the treatment of metals and fibres and surface coating processes etc. For this reason vessels of this type have been given special attention. Other shapes are also included by reference rather than by a worked example.

In explanation of the underlying theory a number of fully worked out examples are given showing the procedure which may be adopted when preparing the necessary calculation sheets for different types of vessels. Such calculation sheets may frequently be required for approval by Inspection and Insurance companies as well as Certification Authorities.

There are no national or international standards or codes of practice that will cover all of the types. Here ASME VIII, Div.1, Appendix 13[1] probably offers the best guidance on a number of different designs. Unfortunately, in its present form, it is rather cumbersome and requires considerable time for proper understanding and assimilation. Another useful source of information on rectangular tanks can be found in the Theory and Practical Design of Bunkers[2]. Rectangular section headers are also covered by the Swedish Pressure Vessels Code[3], British Standard BS 1113: 1969[4] and the Italian Standard ANCC—VSR Collection Section VSR IS: 1978[5]. The last two references need to be viewed with considerable reservations as they appear to contain a number of discrepancies which are inconsistent with the fundamental theory.

Where there is no relevant code, the procedure outlined in this article follows the same logic, based on fundamental engineering theory as used in the codes and should therefore be equally acceptable. Such procedure should be regarded as evidence of good modern day general engineering practice in this field. It is hoped that it will promote a better understanding of the problems associated with such vessels which are often either ignored or not given the consideration and attention which they deserve. Such tanks can be quite complex in their detailed design and unawareness on the part of the designer and/or fabricator, to appreciate the various aspects can lead to costly ramifications later on. These tanks although they appear to be very simple indeed, can nonetheless cause considerable embarrassment if not assessed adequately at the outset.

Additional guidance is given on square/rectangular ducting. Normally such ducting is restricted to 20 psig (0·138 N/mm²). However, the procedure outlined in this article has no limitation per se.

Comparison of the rectangular vessels with the equivalent size cylindrical (circular cross-section) vessels indicate that the former are rather inefficient. Cylindrical vessels will sustain considerably higher pressures, for the same wall thicknesses and size, see Fig. 20. However, practical consideration will often force the designer to select a rectangular shape as the best available option.

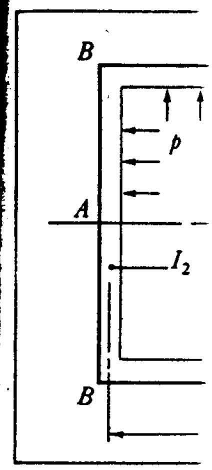
The fundamental theory is applicable to both external and internal pressure. Worked examples given in the text refer to internal pressure for the simple reason that, for the external pressure application, considerable gaps still exist in the knowledge of the allowable compressive stress levels which will not cause buckling or plastic collapse in rectangular and other non-circular tanks. In such instances it should be possible to use the design data contained in the British Standard BS 449: 1969[6] for checking the main stiffeners and beams.

Fundamental theory for rectangular section pressure vessels

Figure 1 shows the basic geometry of the rectangular vessel with sharp corners and which is subjected to a uniform pressure of p.

where L = the longer span
h = the shorter span
I_1 = second moment of area of the beam BCB about its neutral axis
I_2 = second moment of area of the beam BAB about its neutral axis

Due to symmetry about axes AA and CC it will be convenient to



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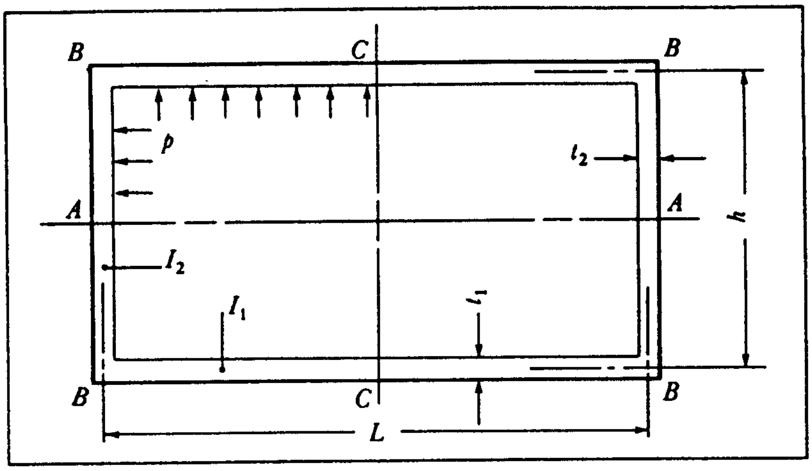

Fig. 1.

analyse one quadrant only of the cross-section shown. This quadrant is in equilibrium under the action of the loads and moments indicated in Fig. 2.

Clearly from the balance of the horizontal and vertical forces acting on the quadrant we obtain

$$T_C = \frac{ph}{2},$$

which represents the direct tensile load in member BC, and

$$T_A = \frac{pL}{2},$$

the tensile load in member AB respectively. In evaluating these tensile forces the thicknesses t_1 and t_2 are considered to be negligible in comparison with dimensions such as L and h, i.e.

$$\left(\frac{L}{2} - \frac{t_2}{2}\right) \text{ effectively equals to } \frac{L}{2}$$

In beams and frames having rigid joints, as in this particular case, the strain energy due to the direct and shear forces is so small in comparison with that due to bending that only the latter need to be considered when evaluating statically indeterminate moments.

In any member of a structure subjected to bending the total strain energy is given by

$$U = \int_0^l \frac{M^2\,dx}{2EI} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where M is the bending moment at any point on the member caused by the combined effect of the imposed loads and the supporting forces and moments, whether statically determinate or not. The integration must be taken over the entire length of each member, of which dx is an element of length.

Two further postulates (Castigliano's theory) help to solve the problem. These are:

(i) The partial differential coefficient of the strain energy in a structure with respect to a load F acting on a structure, is equivalent to the displacement of F along its line of action, i.e.

$$\frac{\partial U}{\partial F} = \int_0^l \frac{2M\,\partial M}{2EI\,\partial F}\,dx$$

$$= \int_0^l \frac{M\,\partial M}{EI\,\partial F}\,dx = \delta \quad \ldots\ldots\ldots (2)$$

(ii) The partial differential coefficient of the strain energy with respect to a moment acting on a structure is equivalent to the angle through which that portion of the structure rotates when the moment is applied

$$\frac{\partial U}{\partial M_x} = \int_0^l \frac{M\,\partial M}{EI\,\partial M_x}\,dx = \phi \quad \ldots\ldots (3)$$

When, as in this case, the support of the structure (point A) does not give way under the action of the loading, then there is no deformation of the structure at this point of support and the two expressions just quoted can be equated to zero.

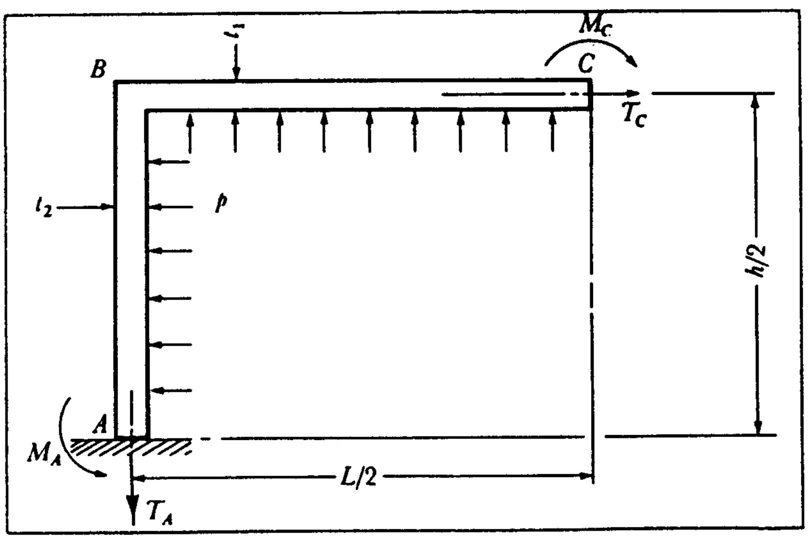

Fig. 2.

By setting down the equation for moments along AB and BC and by considering the strain energy due to bending (by integrating along AB and BC respectively) it can be shown that the moments at the three important points A, B and C become for a general case

$$M_A = \frac{pL^2}{8} \times \left[\beta^2 - 1 + \frac{1}{3}\left(\frac{K+3-2\beta^2}{K+1}\right)\right] \quad \ldots(4)$$

$$M_B = \frac{pL^2}{8} \times \left[1 - \frac{1}{3}\left(\frac{K+3-2\beta^2}{K+1}\right)\right] \quad \ldots\ldots(5)$$

$$M_C = \frac{pL^2}{24}\left(\frac{K+3-2\beta^2}{K+1}\right) \quad \ldots\ldots(6)$$

where

$$K = \frac{I_2}{I_1}\frac{h}{L} \quad \text{and} \quad \beta = \frac{h}{L}$$

Notice that

$$M_B = \frac{pL^2}{8} - \frac{pL^2}{24}\left(\frac{K+3-2\beta^2}{K+1}\right)$$

where the first term denotes the bending moment at mid span for a simply supported beam BCB under the action of uniform load p.

For a uniform wall thickness throughout, the parameter

$$K = \frac{h}{L},$$

which is the same as β and the three moment expressions simplify to the following

$$M_A = \frac{pL^2}{8} \times \left[\beta^2 - 1 + \frac{1}{3}\left(\frac{\beta+3-2\beta^2}{\beta+1}\right)\right] \quad \ldots(7)$$

$$M_B = \frac{pL^2}{8} \times \left[1 - \frac{1}{3}\left(\frac{\beta+3-2\beta^2}{\beta+1}\right)\right] \quad \ldots\ldots(8)$$

$$M_C = \frac{pL^2}{24}\left(\frac{\beta+3-2\beta^2}{\beta+1}\right) \quad \ldots\ldots(9)$$

where once again

$$\beta = \frac{h}{L},$$

the ratio of shorter to longer spans.

By substituting specific values for the parameter β (i.e. $\beta = 0, 0{\cdot}1, 0{\cdot}2, \ldots$ etc. up to $\beta = 1$) we can express the three moments in a very much simplified form

$$M_A = \alpha_A pL^2 \quad \ldots\ldots(10)$$

$$M_B = \alpha_B pL^2 \quad \ldots\ldots(11)$$

$$M_C = \alpha_C pL^2 \quad \ldots\ldots(12)$$

where α_A, α_B and α_C are the three new parameters which, for uniform wall thickness throughout, are dependent on

$\frac{h}{L}$ ratio only.

The plots for these three parameters are shown in Fig. 3, where after simplification these can be written as

$$\alpha_A = \frac{\beta^2 + 2\beta - 2}{24} \quad \ldots\ldots(13)$$

$$\alpha_B = \frac{\beta^2 - \beta + 1}{12} \quad \ldots\ldots(14)$$

$$\alpha_C = \frac{-2\beta^2 + 2\beta + 1}{24} \quad \ldots\ldots(15)$$

Identical plots to those shown in Figure 3 were obtained from the equations given in ASME VIII, Appendix 13[1] and the Swedish Pressure Vessels code[3] indicating that these are also based on fundamental engineering theory.

When comparing the above equations with the corresponding parameters given in these two codes it must be borne in mind that the latter[3] specifies the two spans as $2m \times 2n$ so that the relevant constants α will differ by a factor of four, since $M = \alpha_1 pL^2$ in reference (1) and $M = \alpha_3 pm^2$ in reference (3). Thus for consistency $\alpha_1 L^2$ must be equal to $\alpha_3 m^2$. As $L^2 = 4m^2$ hence this factor of 4 is contained in the parameter α_3 in all the expressions for moment given as $M = \alpha pm^2$.

Hence it can be seen that the approach presented in this article will satisfy both ASME VIII, Div.1 and the Swedish Pressure Vessels code for the plain rectangular vessels but reference has still to be made to these codes for the allowable design stress level and the weld factor where necessary.

From the plots shown in Figure 3 the moment distribution curve along each member can be quite easily obtained by the following method.

(a) For members BCB, span L

First draw to a suitable scale the free end moment distribution curve BCB which is given by the standard equation

$$M_{xb} = \tfrac{1}{2}pL\left\{x - \frac{x^2}{L}\right\} \quad \ldots\ldots(16)$$

where x is the distance from point B. (Distance B–B to represent the span length L.) Refer back to Fig. 3 to obtain the relevant bending moment at the corners, $M_B = \alpha_B pL^2$. Draw a new "zero bending moment" axis 0–0 at a distance equal to M_B below the original datum line BB (as shown in Fig. 4a). The resultant sketch will give the complete bending moment distribution diagram for the longer span length L. Moment at any point along BB is then simply given by the vertical intercept, either above or below

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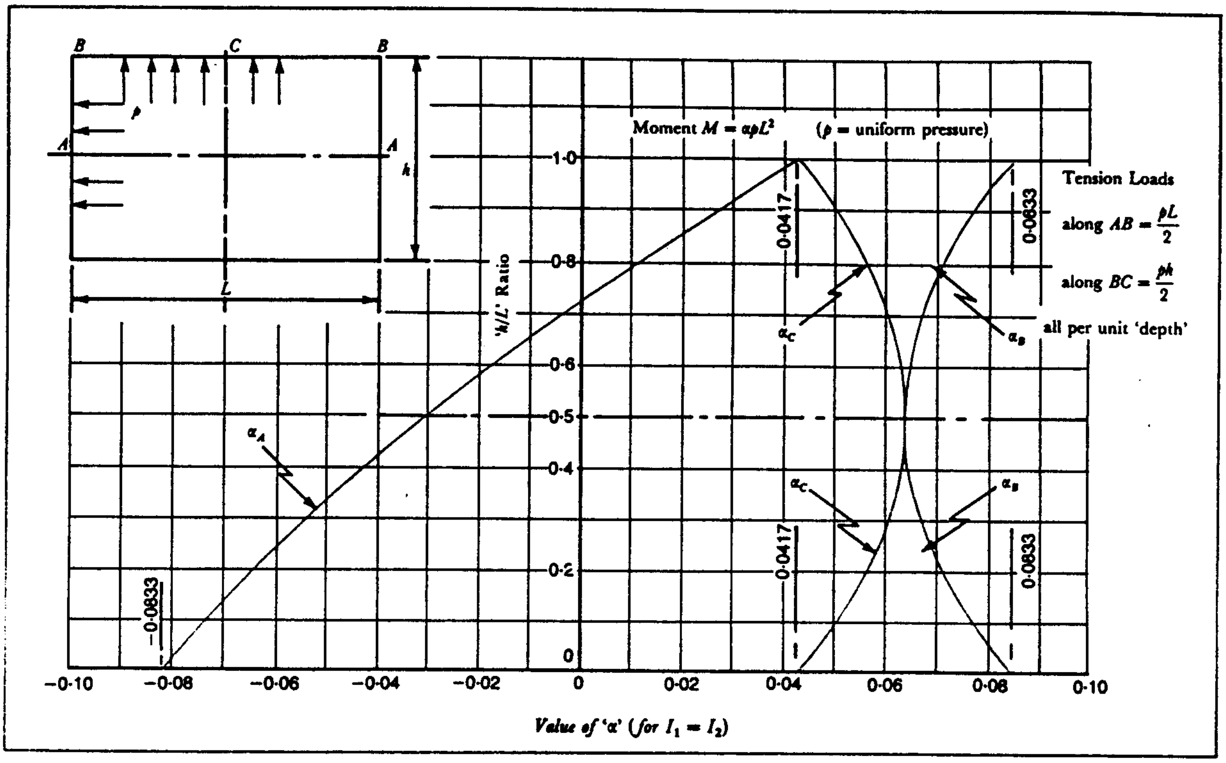


Fig. 3.

the new datum line 0–0 as the case may be. In this particular instance Fig. 4 shows the moment distribution curve which is applicable to the following three geometries: (i) datum line 0–0 for a rectangular header whose h/L ratio equals 0·5, (ii) datum line 0′–0′ for square header, i.e. $h = L$ and (iii) for built in beam where $h = o$. The points of contraflexure are also shown for these cases. This sort of information could prove useful when the decision has to be made on the best location of the welded seam or any other outside attachment.

(b) For members *BAB*, span *h*

Similar procedure to that described above can be used to obtain the moment distribution diagram for the shorter span. The only difference in procedure is that the initial free end bending moment curve is now given by the equation

$$M_{xa} = \frac{1}{2}ph\left\{x - \frac{x^2}{h}\right\} \quad \ldots\ldots\ldots(17)$$

where x is the distance from point B (towards A this time). The new distance BB should now represent, to the same scale as above, the shorter span h.

The basic engineering theory and the above procedure indicate that each member of a rectangular section vessel can be treated as an initially simply supported (free end) beam uniformly loaded along its entire length which is then subjected to the end moments M_B, the latter determined from Fig. 3. This approach will be useful for calculating the central deflection of the members. This is illustrated in Figs. 4(b) and (c) and the plot for the central deflection of the longer span L is given in Fig. 5.

So far we have dealt essentially with a uniform wall rectangular vessels. The preceding basic theory is equally applicable to rectangular vessels which have peripheral stiffeners spaced along the length of the vessel as shown in Fig. 6.

In such cases we have to check not only the strength of the stiffeners but also the stress levels in the wall panels between these stiffeners.

The strength of the peripheral stiffeners can be determined by the method described above, as for the plain rectangular vessels, by substituting ps for the uniform pressure load p used in the preceding analysis. Equations (4), (5) and (6) can be used directly for a general case where the second moments of area of the stiffeners I_1 and I_2 and the wall thicknesses t_1, t_2 of the two main sides are

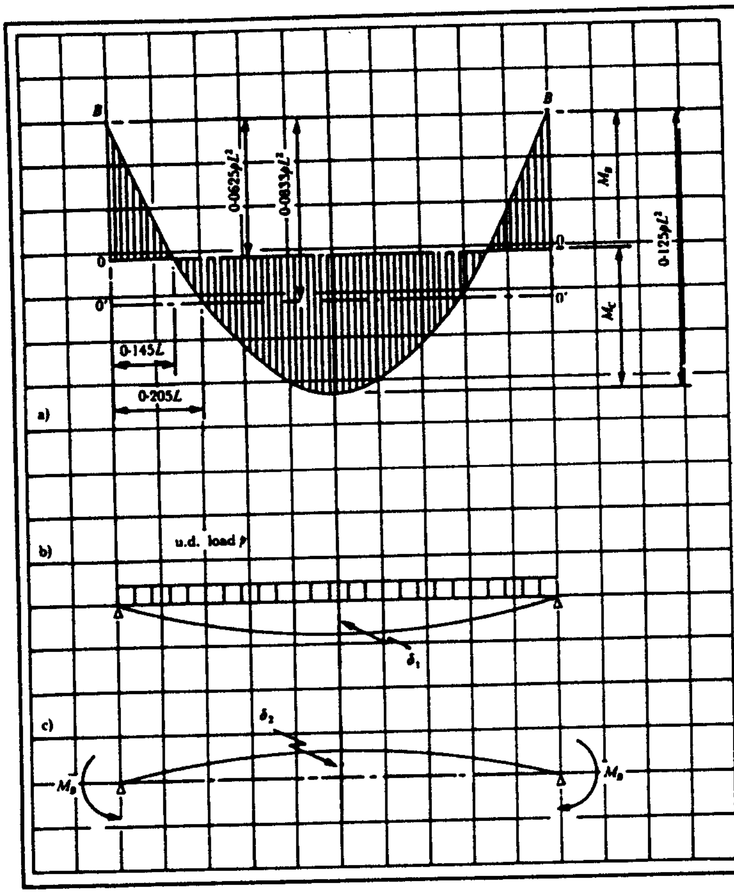


Fig. 4.

Fig. 4. Central deflections

$$\delta_1 = \frac{5pL^4}{384EI} = \frac{L^2}{48EI}(5M_C) \quad \text{as} \quad M_C = \frac{pL^2}{8}$$

$$\delta_2 = \frac{M_B L^2}{8EI} = \frac{L^2}{48EI}(6M_B)$$

$$\therefore \delta_1 - \delta_2 = \frac{L^2}{48EI}(5M_C - 6M_B)$$

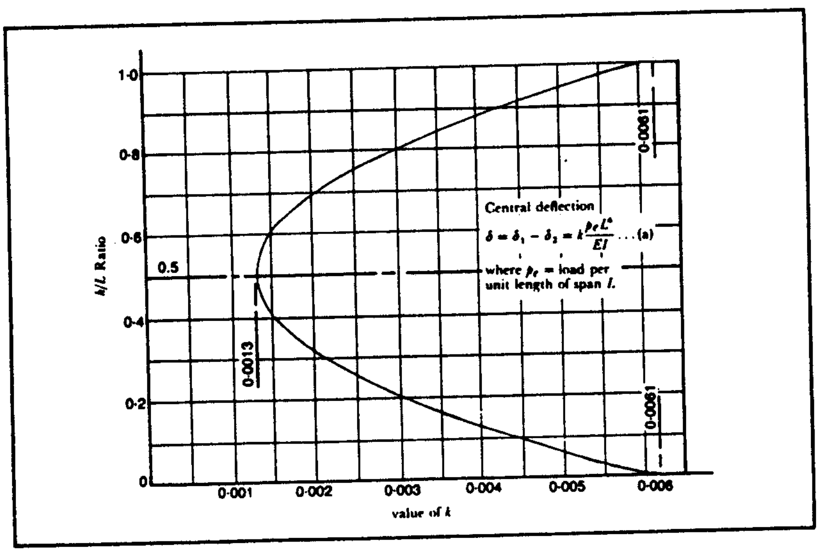


Fig. 5.

different. For uniform wall and stiffener sections Equations (10), (11) and (12) and Fig. 3 become once more applicable provided ps is substituted for p in the relevant equations.

The wall panels between the stiffeners can be treated as rectangular panels fixed (built-in) at all four edges and subjected to a uniform pressure load p over the entire area. Reference (7) covers this particular case and gives the maximum bending stress, which occurs at the centre of the long edges, as

$$\sigma = \beta \frac{pb^2}{t^2},$$

where the value of β depends solely on the ratio of the two sides a/b, b is the width or the shorter span and t is the panel plate

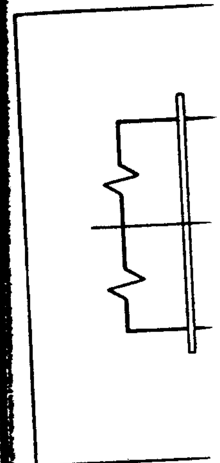
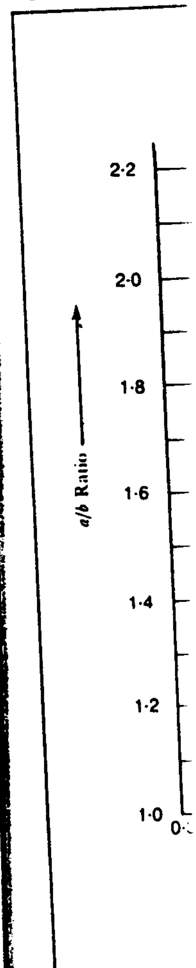


Fig. 6.



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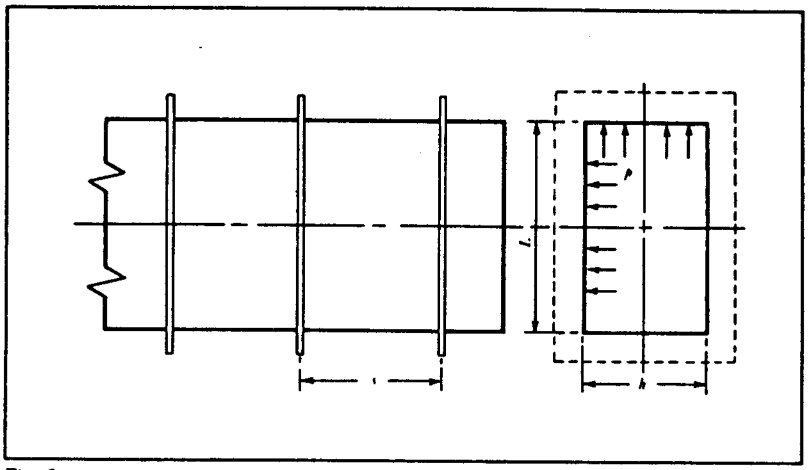

Fig. 6.

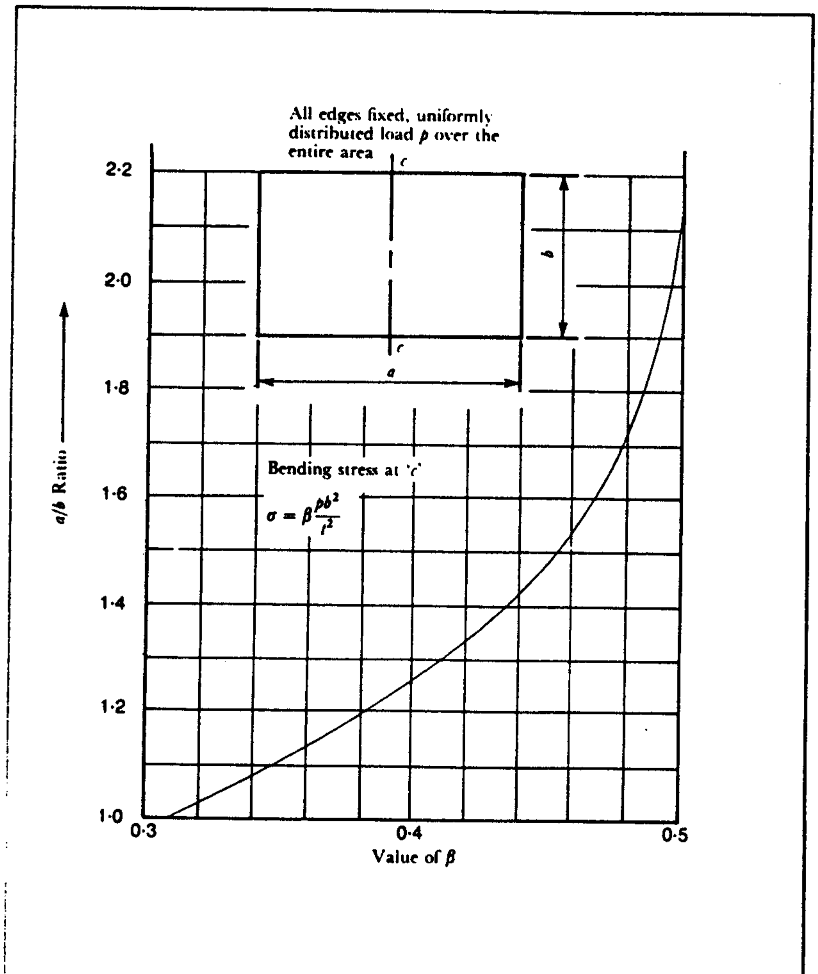

Fig. 7.

thickness. Fig. 7 gives the plot for the variable β for various a/b ratios. Notice that for a/b values above 2·15 the parameter $\beta = 0{\cdot}5$, giving

$$\sigma = 0{\cdot}5\frac{pb^2}{t^2} \quad \text{or} \quad 0{\cdot}5\frac{pL^2}{t^2}.$$

This represents the same situation that occurs for a built-in beam of span b. Here the end moment

$$M_B = \frac{pL^2}{12}$$

and the plate section modules for a unit width strip

$$Z = \frac{t^2}{6}.$$

Hence the bending stress at the built in edge

$$\sigma = \frac{M}{Z} = \frac{6pL^2}{12t^2} = 0{\cdot}5\frac{pL^2}{t^2},$$

i.e. the same as above. This confirms that for wall panels whose a/b ratio exceeds 2·15 we can treat the central portion of such panels as a fixed-in beam of span equal to the width of the panel.

One further detail which will require consideration is the solution for the corner wall panels, whether the corner occurs between the main side panels or between the side panels and the flat ends which may have transverse stiffeners. Such details can be dealt with by evaluating the bending moments and tensile loads shown in Fig. 8. The information presented here is based on the basic theory contained in Reference (8) by combining the two separate loading conditions for panels L and h respectively.

Basic information on critical moments and tensile loads is also given for

(a) rectangular vessels with radiused corners—see Fig. 9;
(b) elliptical vessels—see Fig. 10;

(c) oblong vessels of uniform thickness—see Figs. 11 and 12.

Table 1 in the Appendix gives some basic equations for the simple geometries and loading systems considered in this article.

WORKED EXAMPLES

1. Open top rectangular tank with continuous horizontal wall stiffeners

Figure 13 shows the essential details of one such tank measuring 5500 × 2000 × 2500 mm deep. The tank is to contain liquid of specific density 1·5. It is to be supported on beam members forming part of the general plant structure.

The tank is to be built from 6-mm thick plate material of 432 N/mm² ultimate strength. The corrosion effect on the plate thickness is considered to be negligible during the useful life of the tank.

It would normally require several attempts to establish the optimum size of the stiffeners and their respective spacing. The following check will deal with the tank as shown in Fig. 13 in order to demonstrate the design method rather than the final choice.

The pressure distribution on the tank walls will be linear and as shown in Fig. 14. The pressure at the bottom of the tank due to the 2·5 m head of liquid of specific density of 1·5 will be

$$p = \frac{1\cdot5 \times 2\cdot5}{10} = 0\cdot375 \text{ kg/cm}^2$$

(as 10 m head of water is equivalent to 1 atü or 1 kg/cm² pressure), or in Newtons per mm² this pressure is equivalent to

$$p = \frac{0\cdot375 \times 9\cdot81}{100}$$

$= 0\cdot0368$ N/mm² or $3\cdot68 \times 10^{-2}$ N/mm².

(A) Check on Stiffeners

(i) Considering the first stiffener from the bottom, namely *S*1. It is fabricated from 200 × 100 × 8 mm rectangular hollow section whose properties are as follows

I_{xx} = moment of inertia = 2269 cm⁴

Z_{xx} = elastic modulus = 227 cm³

A = sectional area = 45·1 cm²

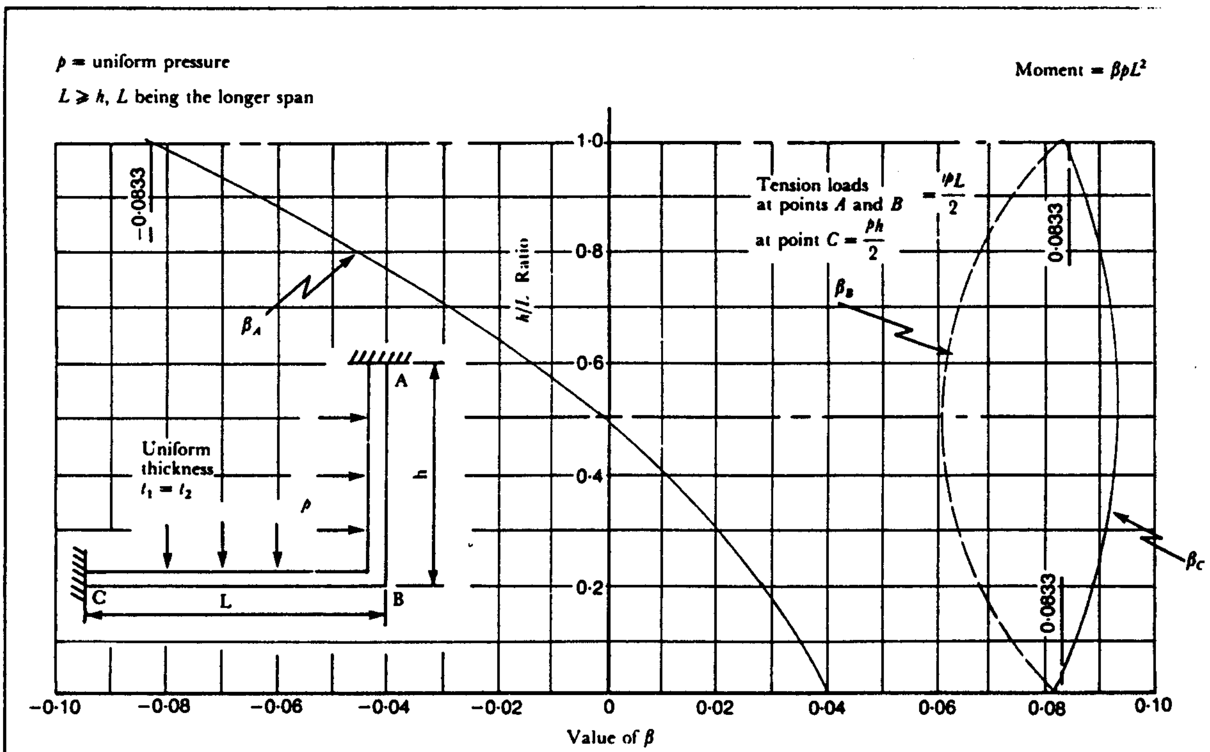

Fig. 8.

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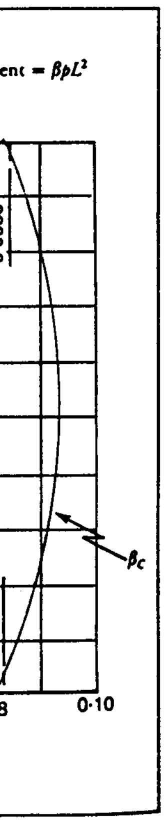

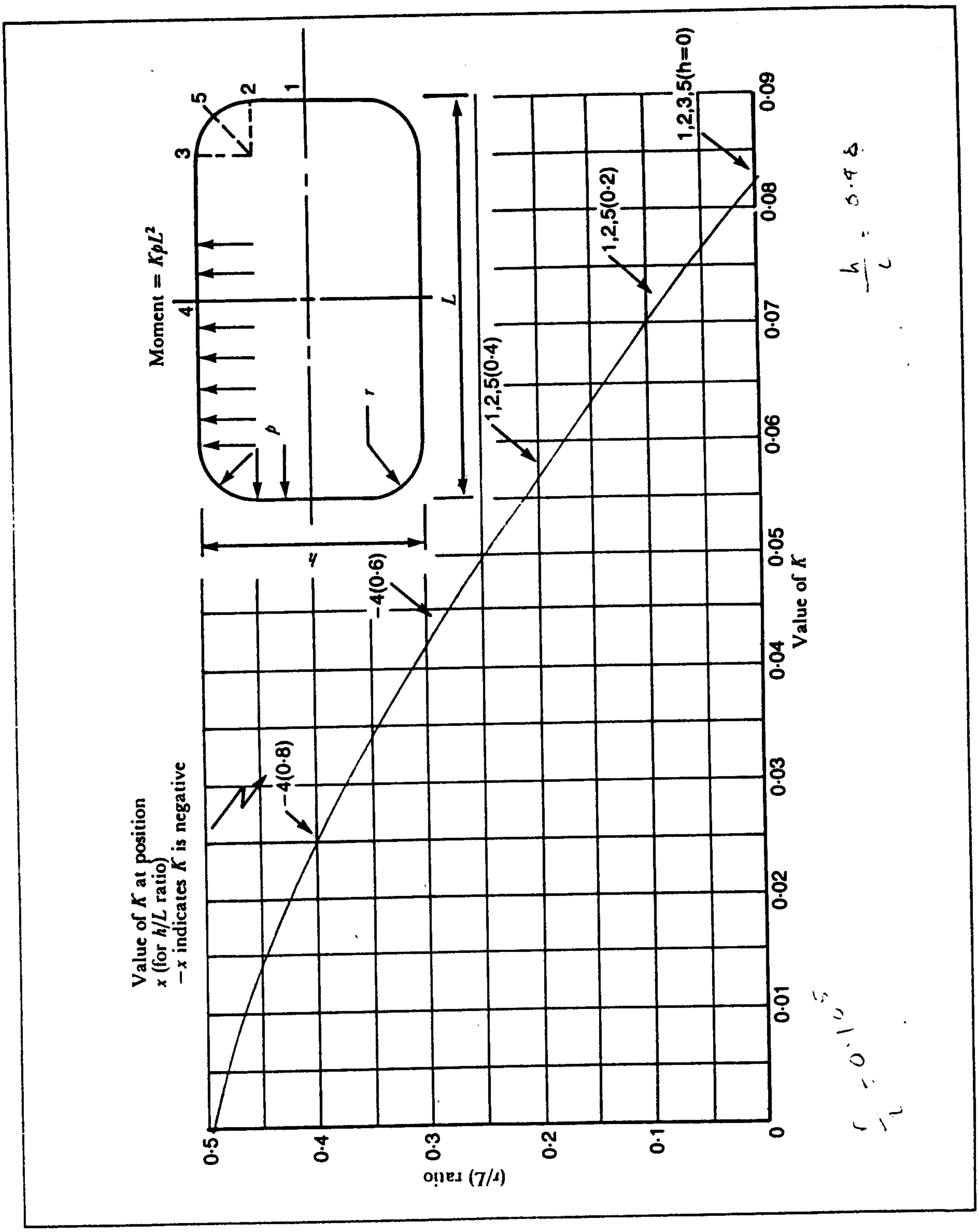

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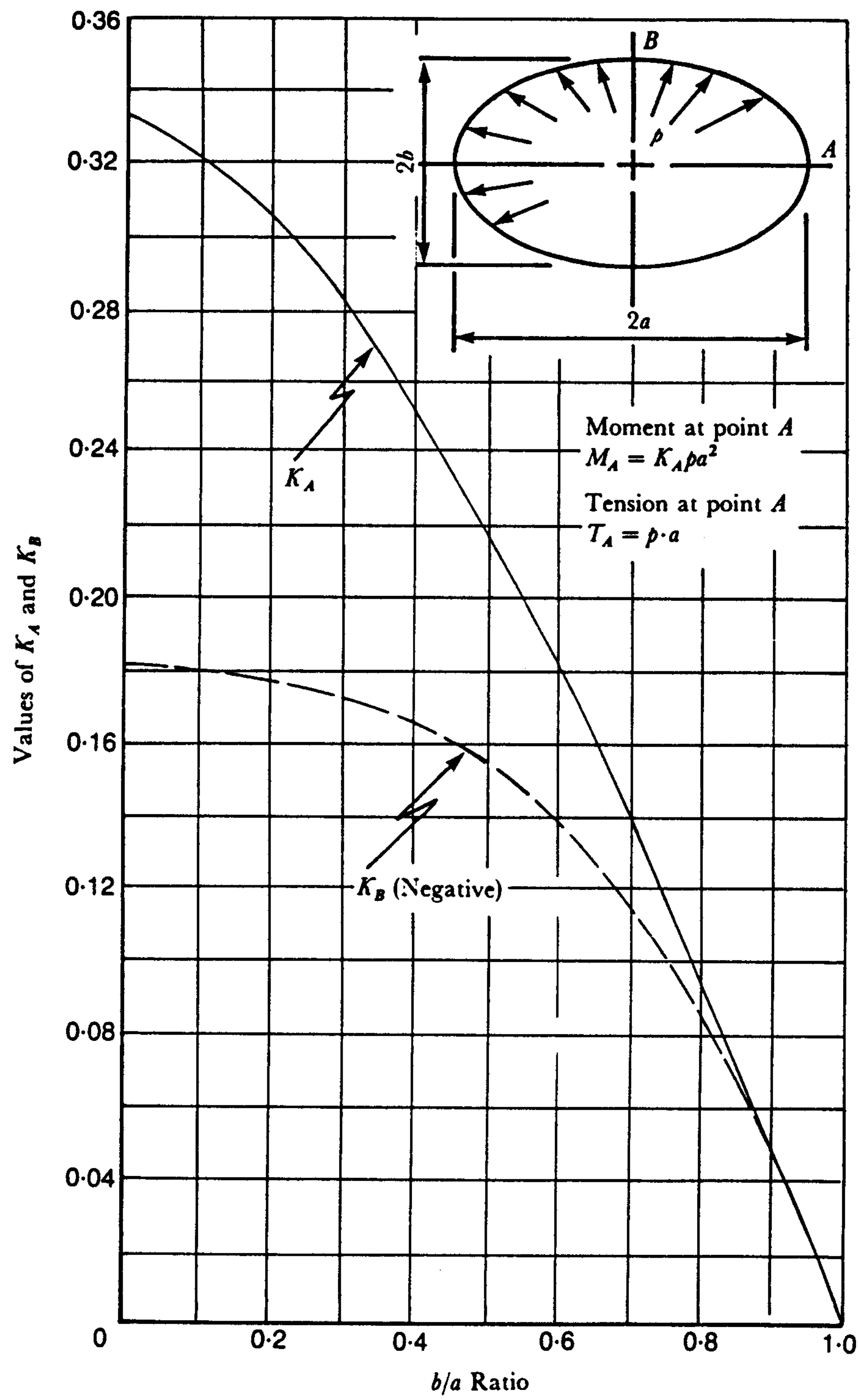

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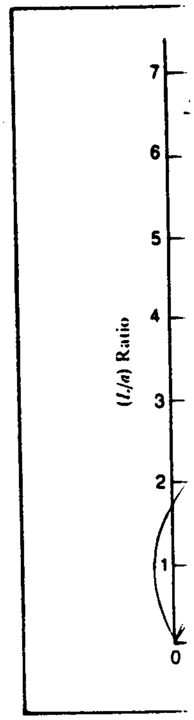

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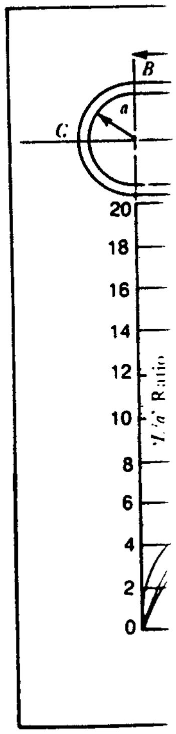

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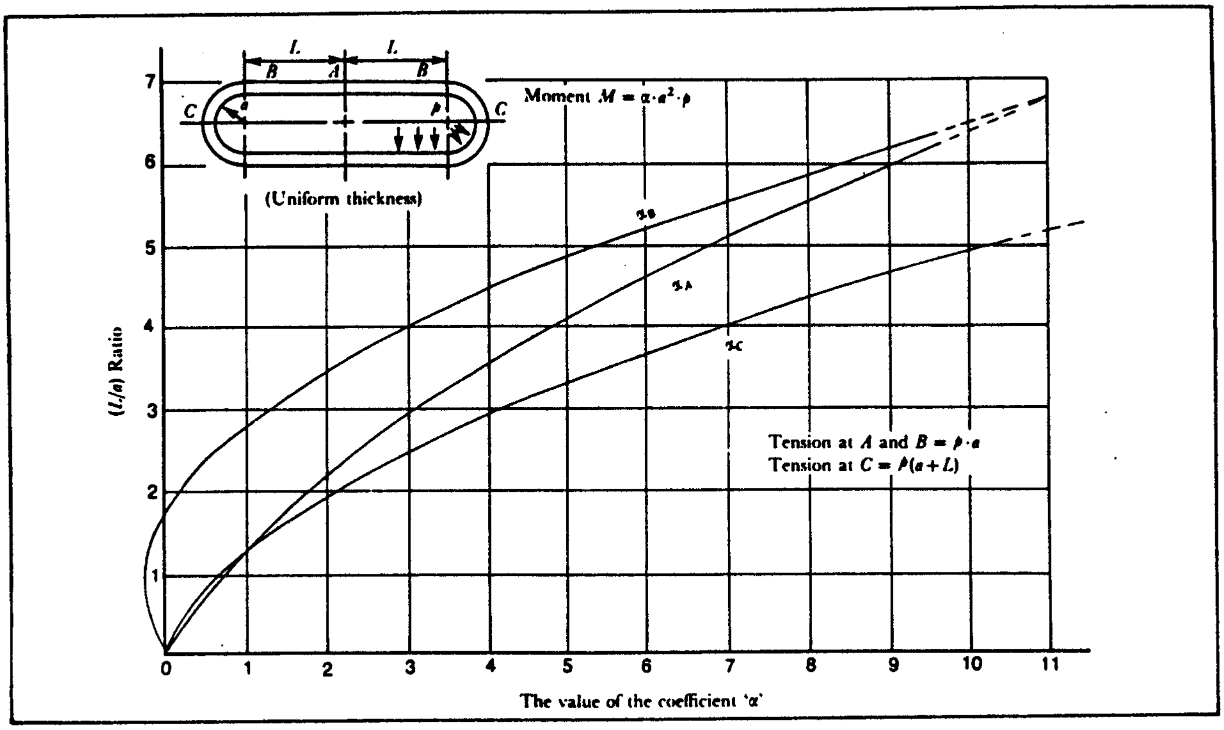

L
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B
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B
C
C
a
p
(Uniform thickness)
Moment M = α·a²·p
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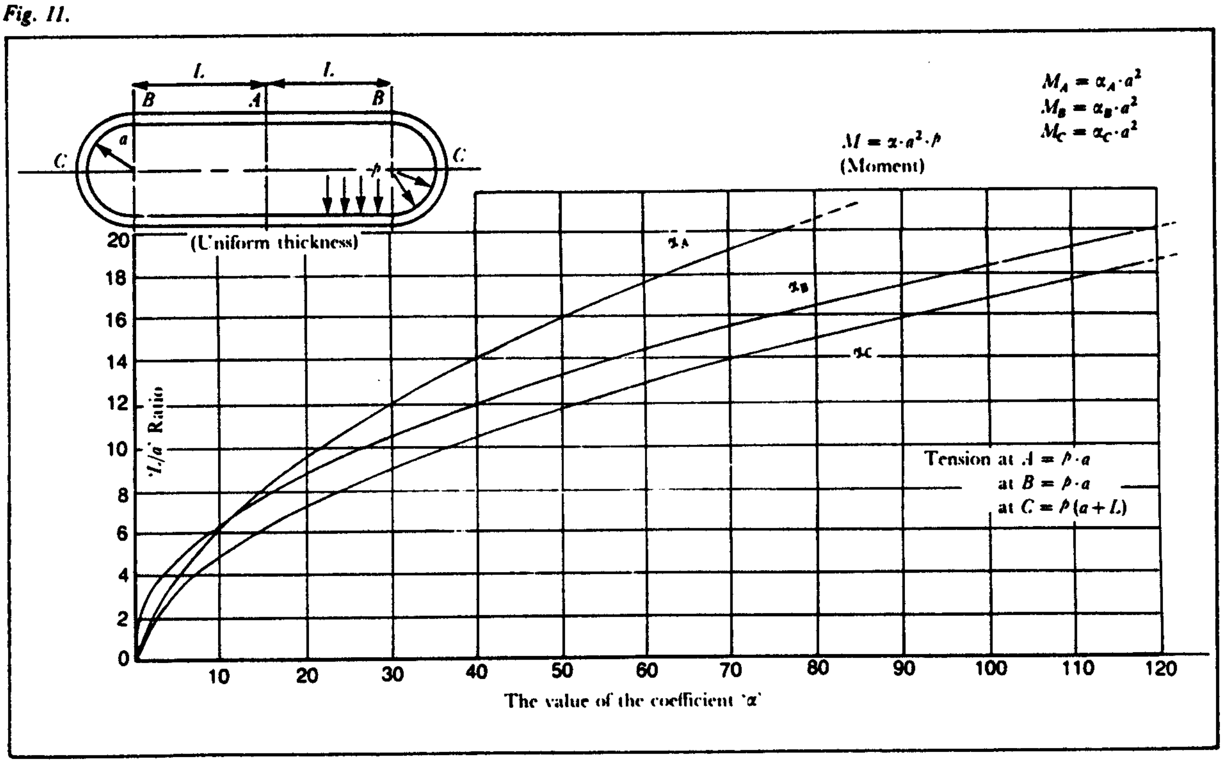
αA
αC
Tension at A and B = p·a
Tension at C = p(a+L)
(L/a) Ratio
The value of the coefficient 'α'

Fig. 11.



L
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C
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(Uniform thickness)
M = α·a²·p
(Moment)
M_A = α_A·a²
M_B = α_B·a²
M_C = α_C·a²
αA
αB
αC
Tension at A = p·a
at B = p·a
at C = p(a+L)
'L/a' Ratio
The value of the coefficient 'α'

Fig. 12.

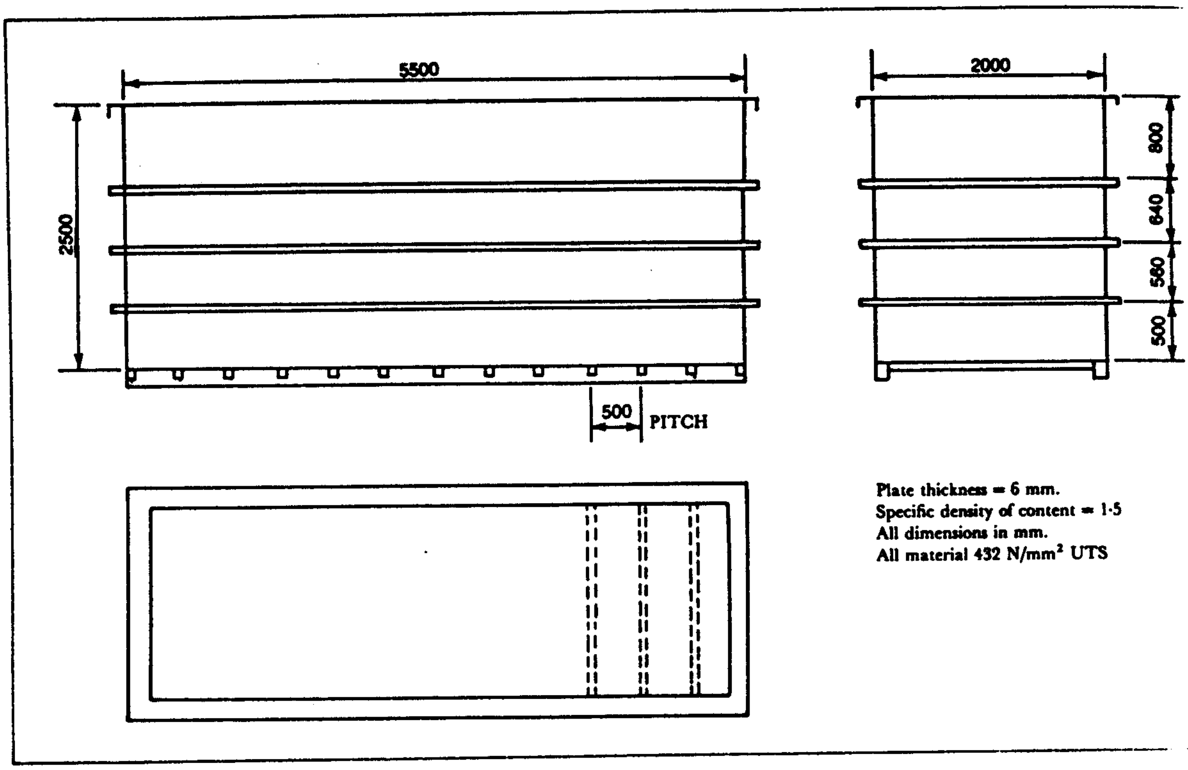

Fig. 13.

The pressure load on this stiffener per mm of length (span) will be, according to Figure 14,

$$p_1 = \tfrac{1}{2}(2{\cdot}52+3{\cdot}32) \times 10^{-2} \times \left(\frac{500+560}{2}\right)$$
$$= 15{\cdot}476 \text{ Newtons per mm of span.}$$

From Fig. 3, for the h/L ratio of

$$\frac{2000}{5500} = 0{\cdot}364,$$

the max. bending moment M_B is given as

$$M_B = 0{\cdot}0638 p_1 L^2$$
$$= 0{\cdot}0638 \times 15{\cdot}476(5500)^2$$
$$= 29{\cdot}87 \times 10^6 \text{ N.mm}$$

Since the section modulus Z of this stiffener = 227 cm³ (or 227 × 1000 mm³) hence the bending stress at the corners of the stiffener will be

$$\sigma_B = \frac{29{\cdot}87 \times 10^6}{227 \times 1000}$$
$$= \underline{131{\cdot}6 \text{ N/mm}^2}$$

In addition there will be a direct tensile load acting on the stiffener. This tensile load can be calculated as follows

$$T_B = \frac{p_1 \times L}{2} = \frac{15{\cdot}476 \times 5500}{2}$$
$$= 42559 \text{ Newtons}$$

Hence the direct tensile stress at corners (or along the shorter spans) is given by

$$\sigma_D = \frac{42559}{45{\cdot}1 \times 100} = \underline{9{\cdot}44 \text{ N/mm}^2}$$

Thus the combined maximum tensile stress in the stiffener $S1$ is given by

$$\sigma_T = \sigma_B + \sigma_D = 131{\cdot}6 + 9{\cdot}44$$
$$= \underline{141{\cdot}04 \text{ N/mm}^2}$$

(ii) For stiffener $S2$, fabricated from the same hollow section, the corresponding moment, tensile force and stress levels were found to be as follows

The pressure load

$$p_2 = \tfrac{1}{2}(1{\cdot}64+2{\cdot}52) \times 10^{-2} \times \left(\frac{560+640}{2}\right)$$
$$= 12{\cdot}48 \text{ Newtons per mm of span.}$$

The max. bending moment

$$M_B = 0{\cdot}0638 \times 12{\cdot}48 \times (5500)^2$$
$$= 24{\cdot}086 \times 10^6 \text{ N.mm.}$$

The bending stress σ_B

$$= \frac{24{\cdot}086 \times 10^6}{227 \times 1000} = 106{\cdot}1 \text{ N/mm}$$

The direct tensile force

$$T_B = \frac{12{\cdot}48 \times 5500}{2}$$
$$= 34320 \text{ Newtons}$$

So that the direct

$$\sigma_D = \frac{34320}{45{\cdot}1 \times 100} =$$

Thus the combin
stress in stiffener S

$$\sigma_T = 106{\cdot}1 + 7{\cdot}6$$
$$= \underline{113{\cdot}7 \text{ N/mm}}$$

It appears that a l
stiffener section co
at this position, su
200 × 100 × 6·3 m
which will result i
bending stress lev

$$\sigma_T = \frac{24{\cdot}086 \times 10}{185 \times 100}$$
$$= 130{\cdot}2 + 9{\cdot}5$$

(iii) Stiffener S
150 × 100 × 6·3 m

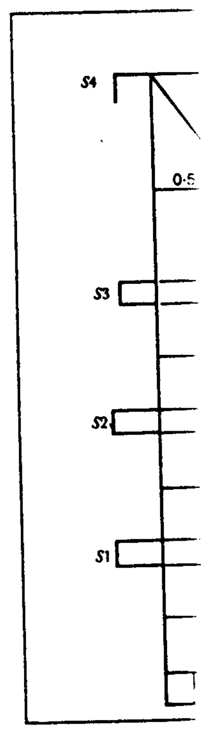

Fig. 14.

So that the direct tensile stress

$$\sigma_D = \frac{34320}{45 \cdot 1 \times 100} = 7 \cdot 6 \text{ N/mm}^2$$

Thus the combined max. tensile stress in stiffener $S2$ is given by

$$\sigma_T = 106 \cdot 1 + 7 \cdot 6$$
$$= \underline{113 \cdot 7 \text{ N/mm}^2}$$

It appears that a lighter stiffener section could be used at this position, such as $200 \times 100 \times 6 \cdot 3$ mm hollow section which will result in the combined bending stress level of

$$\sigma_T = \frac{24 \cdot 086 \times 10^6}{185 \times 1000} + \frac{34320}{36 \cdot 0 \times 100}$$
$$= 130 \cdot 2 + 9 \cdot 5 = 139 \cdot 7 \text{ N/mm}^2$$

(iii) Stiffener $S3$, fabricated from $150 \times 100 \times 6 \cdot 3$ mm hollow section.

$$p_3 = \tfrac{1}{2}(0 \cdot 58 + 1 \cdot 64) \times 10^{-2} \times \left(\frac{640+800}{2}\right)$$
$$= 7 \cdot 992 \text{ Newtons per mm span}$$

$$M_B = 0 \cdot 0638 \times 7 \cdot 992 \times (5500)^2$$
$$= 15 \cdot 42 \times 10^6 \text{ N.mm}$$

$$T_B = \frac{7 \cdot 992 \times 5500}{2}$$
$$= 21978 \text{ Newtons and}$$

$$\sigma_D = \frac{21978}{29 \cdot 7 \times 100} = 7 \cdot 4 \text{ N/mm}^2$$

Thus the combined max. tensile stress in stiffener $S3$
$= \underline{134 \cdot 9 \text{ N/mm}^2}$.

(iv) Stiffener $S4$, the top flange, fabricated from $100 \times 50 \times 6$ mm channel section, $Z_{xx} = 41 \text{ cm}^3$ and $A = 13 \cdot 2 \text{ cm}^2$.

$$p_4 = \tfrac{1}{2} \times 0 \cdot 58 \times 400$$
$$= 11 \cdot 6 \text{ Newtons per mm span}$$

$$M_B = 0 \cdot 0638 \times 11 \cdot 6 \times (5500)^2$$
$$= 22 \cdot 387 \times 10^6 \text{ N.mm}$$

$$\therefore \sigma_B = \frac{22 \cdot 387 \times 10^6}{41 \times 1000}$$
$$= 54 \cdot 6 \text{ N/mm}^2$$

$$T_B = \frac{11 \cdot 6 \times 5500}{2}$$
$$= 31900 \text{ Newtons, and}$$

$$\sigma_D = \frac{31900}{13 \cdot 2 \times 100} = 24 \cdot 2 \text{ N/mm}^2$$

Hence the total combined tensile stress

$$\sigma_T = 78 \cdot 8 \text{ N/mm}^2.$$

This appears to be rather lightly stressed by comparison with other stiffeners but normally a heavier section is required at the "top flange position" for handling purposes.

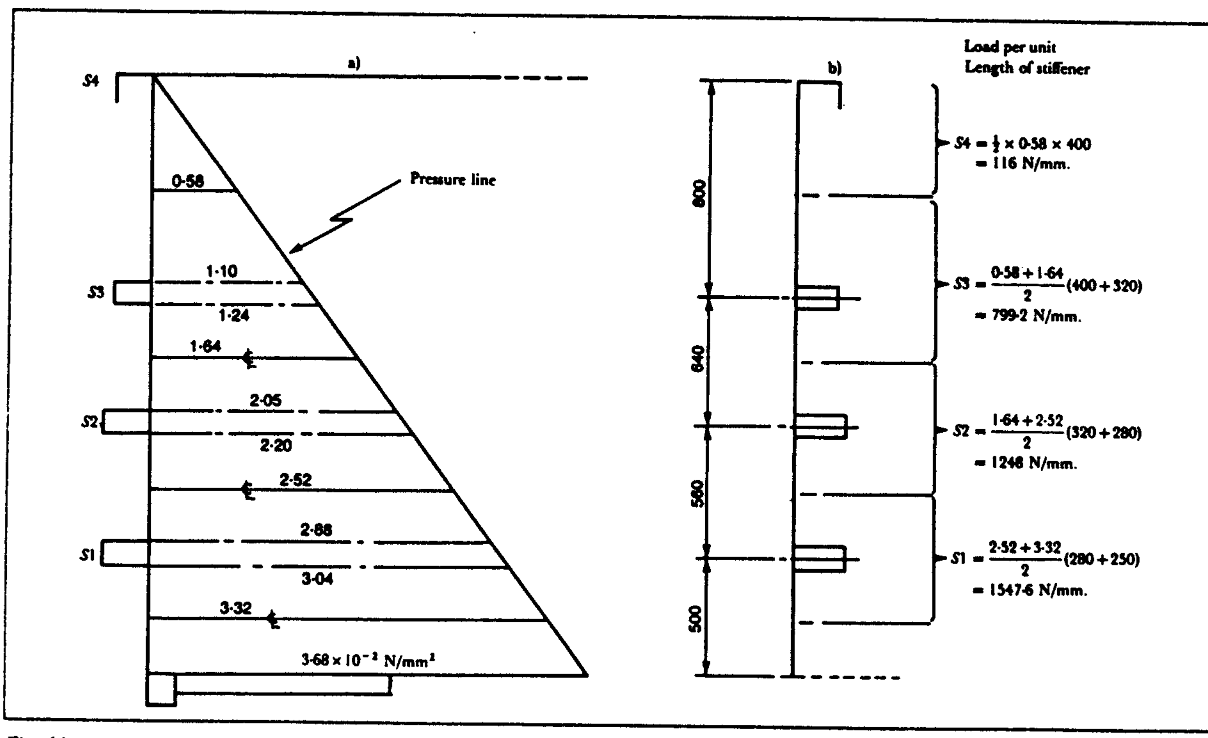

Fig. 14.

B. Check on wall panels

(i) The lowest wall panel (between the floor plate and the edge of stiffener $S1$) is loaded as shown in Figs. 14 and 15. The actual trapezoidal pressure distribution can be substituted by a uniform load case shown in Fig. 15(a) and a triangular load case shown in Fig. 15(b). These two cases can now be evaluated by the basic engineering beam theory.

From the equations contained in Table III of Reference (7) the bending moment at point B will be

$$M_1 = \frac{p_1 L^2}{12} \quad \text{for case (a), and}$$

$$M_2 = \frac{p_2 L^2}{20} \quad \text{for case (b)}$$

giving a combined bending moment at this point

$$M_B = \left(\frac{p_1}{12} + \frac{p_2}{20}\right) L^2$$

Thus, substituting the relevant values we have

$$M_B = \left(\frac{3{\cdot}04}{12} + \frac{0{\cdot}64}{20}\right) \times 10^{-2} \times (450)^2$$
$$= 577{\cdot}8 \text{ N.mm per unit width.}$$

Now the section modulus of a unit width of plate of thickness t is given by

$$Z = \frac{t^2}{6},$$

so that the bending stress at point B becomes

$$\sigma_B = \frac{6M}{t^2} = \frac{6 \times 577{\cdot}8}{6^2}$$
$$= \underline{96{\cdot}3 \text{ N/mm}^2}$$

(ii) Second wall panel, between

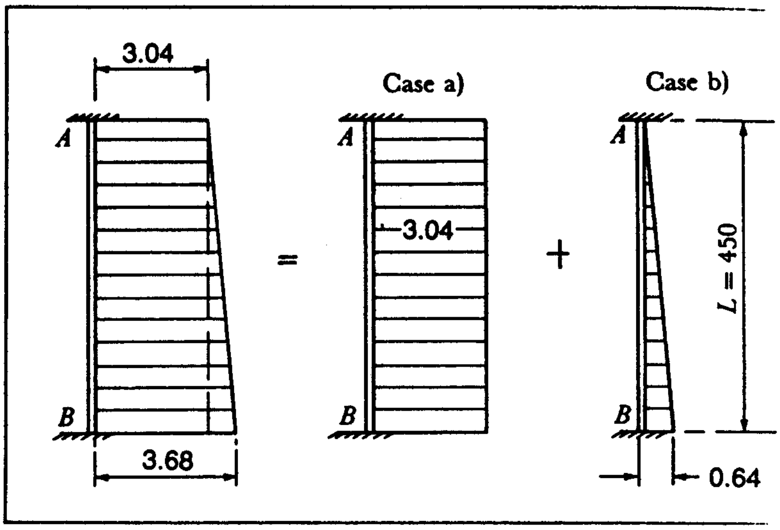

Fig. 15.

edges of stiffeners $S1$ and $S2$, $L = 460$ mm.

$$M_B = \left(\frac{2{\cdot}2}{12} + \frac{0{\cdot}68}{20}\right) \times 10^{-2} \times (460)^2$$
$$= 459{\cdot}9 \text{ N.mm per unit width.}$$

$$\therefore \sigma_B = \frac{6 \times 459{\cdot}9}{6^2}$$
$$= 76{\cdot}65 \text{ N/mm}^2.$$

(iii) Third panel, between edges of stiffeners $S2$ and $S3$, $L = 550$ mm.

$$M_B = \left(\frac{1{\cdot}24}{12} + \frac{0{\cdot}81}{20}\right) \times 10^{-2} \times (550)^2$$
$$= 435{\cdot}1 \text{ N.mm per unit width}$$

and

$$\sigma_B = \frac{6 \times 435{\cdot}1}{6^2}$$
$$= 72{\cdot}52 \text{ N/mm}^2.$$

C. Floor panels and stiffeners

(i) Floor panels, between the horizontal stiffeners which are spaced at 500 mm pitch, see Fig. 13.

Here $b = 500 - 80 = 420$ mm.

Using Fig. 7 for the built in plate panels, where

$$\frac{a}{b} = \frac{2000}{420} = 4{\cdot}76$$

which is greater than 2·15, we have the bending stress at built in edges

$$\sigma_B = 0{\cdot}5 \times \frac{3{\cdot}68 \times 10^{-2} \times 420^2}{6^2}$$
$$= 90{\cdot}2 \text{ N/mm}^2$$

(ii) Floor stiffeners, fabricated from $120 \times 80 \times 6{\cdot}3$ hollow sections, $Z = 74{\cdot}6 \text{ cm}^3$.

Treating the stiffeners as built in beams, the load per beam $p = 3{\cdot}68 \times 10^{-2} \times 500 = 18{\cdot}4$ Newtons per mm of span.

Therefore the bending moment at fixed ends

$$M_B = \frac{pL^2}{12} = \frac{18{\cdot}4 \times 1800^2}{12}$$

and the bending stress

$$\sigma_B = \frac{M}{Z} = \frac{18{\cdot}4 \times 1800^2}{12 \times 74{\cdot}6 \times 1000}$$
$$= \underline{66{\cdot}6 \text{ N/mm}^2}$$

The preceding a
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locations do not
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above are accept
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2. Enclosed Tank with Fl Cover with G

Fig. 16 shows th
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depth of 1·2 m.
pressure = 0·06
Because of th
flanged joint th
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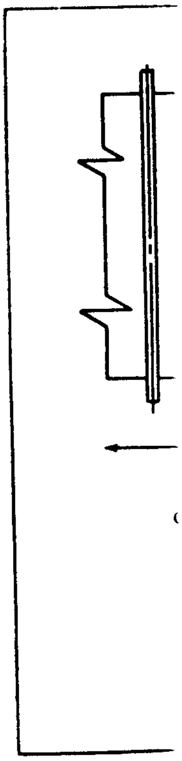
(i) Consider t
the tank itself.



The preceding analysis indicates that the stress values at various locations do not exceed the allowable stress levels confirming that all the details considered above are acceptable and no further refinement is really necessary.

2. Enclosed Rectangular Tank with Flat Bolted Top Cover with Gasket Seal

Fig. 16 shows the essential details of the proposed tank. The tank is to contain liquid of specific density of 1·0 up to a maximum depth of 1·2 m. The "gas space" pressure = 0·069 N/mm² (10 psig).

Because of the presence of the flanged joint the cross-section of the tank cannot be considered as an integral entity. In this instance the cover and the tank have to be treated independently.

(i) Consider the cross-section of the tank itself. The vertical stiffeners on the side wall can be considered as beams built-in at the lower ends and simply supported at the flange face joint level and loaded as shown in Fig. 17.

The pressure at the bottom of the tank will be given by

$$p_2 = 0{\cdot}069 + 0{\cdot}012$$
$$= 0{\cdot}081 \text{ N/mm}^2$$

The pitch between the stiffeners = 440 mm. Thus the loading per unit length of span will be

$$p_1 = 0{\cdot}069 \times 440$$
$$= 30{\cdot}36 \text{ Newtons per unit length of span } AB,$$

and

$$p_2 = 0{\cdot}012 \times 440$$
$$= 5{\cdot}28 \text{ Newtons per unit of span } AB.$$

The two loading conditions shown above will produce the following bending moments at base (point B)

$$M_1 = \frac{p_1 L^2}{8} \text{ for case (a), and}$$

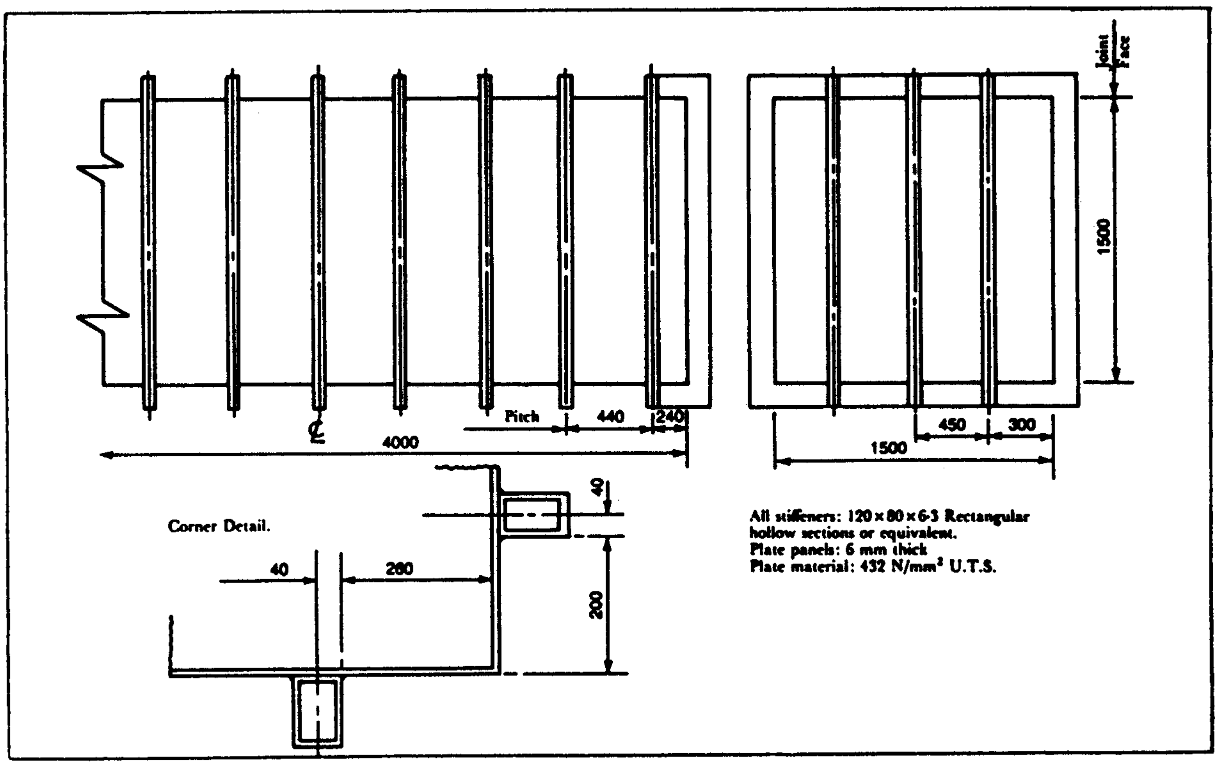
$$M_2 = \frac{p_2 L^2}{15} \text{ for case (b),}$$

assuming that the triangular load distribution is over the entire length of 1·5 m instead of 1·2 m. This will give slightly conservative results.

Thus the combined bending moment at base will be

$$M_B = M_1 + M_2$$
$$= \frac{30{\cdot}36}{8} \times 1500^2 + \frac{5{\cdot}28}{15} \times 1500^2$$
$$= 9330750 \text{ N.mm}$$

Hence the bending stress at this location



$$\sigma_B = \frac{M}{Z} = \frac{9330750}{74{\cdot}6 \times 1000}$$
$$= \underline{125{\cdot}08\ \text{N/mm}^2}$$

In addition there will be a tensile load equal to

$$\frac{p_2 L}{2}$$

acting on each stiffener. Therefore

$$\sigma_D = \frac{0{\cdot}081 \times 440 \times 1500}{2 \times 23{\cdot}4 \times 100}$$
$$= 11{\cdot}42\ \text{N/mm}^2$$

So that the total tensile stress

$$\sigma_T = 125{\cdot}08 + 11{\cdot}42$$
$$= 136{\cdot}50\ \text{N/mm}^2.$$

(ii) Consider the floor panels. The space between edge of stiffeners, b

$$= 440 - 80 = 360\,\text{mm} = b.$$

The length of these panels $a = 1500$, so that

$$\frac{a}{b} = \frac{1500}{360} = 4{\cdot}17$$

which again is greater than 2·15. Hence from Fig. 7 the bending stress at built-in edges is given by

$$\sigma_B = 0{\cdot}5p\left(\frac{L}{t}\right)^2.$$

Thus

$$\sigma_B = 0{\cdot}5 \times 0{\cdot}081\left(\frac{360}{6}\right)^2$$
$$= 145{\cdot}8\ \text{N/mm}^2.$$

The longitudinal direct tensile load =

$$\frac{0{\cdot}081 \times 1500}{4}$$

giving a direct tensile stress

$$\sigma_D = \frac{0{\cdot}081 \times 1500}{4 \times 6}$$
$$= 5{\cdot}06\ \text{N/mm}^2.$$

Hence the combined tensile stress at the built-in edges of the floor panels is

$$\sigma_T = \underline{150{\cdot}86\ \text{N/mm}^2}.$$

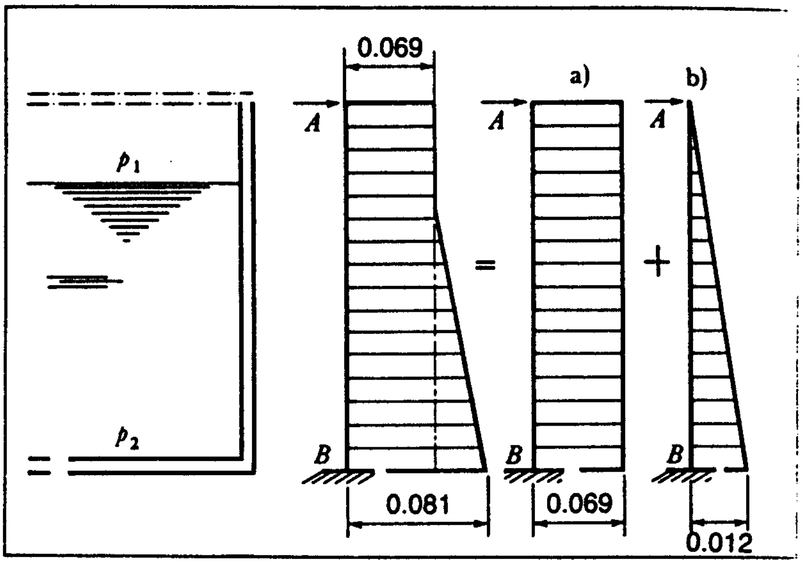


Fig. 17.

(iii) Consider now the corner wall panels (see corner detail in Fig. 16). Here $h = 200$ mm and $L = 260$ mm, $p = 0{\cdot}081$ N/mm². Therefore

$$\frac{h}{L} = \frac{200}{260} = 0{\cdot}769$$

and from Fig. 8, $\beta_C = 0{\cdot}161$, so that the bending moment at point C, see sketch – Fig. 18, will be:

$$M_C = 0{\cdot}091 \times 0{\cdot}081 \times 260^2$$
$$= 498{\cdot}3\ \text{N.mm per unit width}$$

Hence

$$\sigma_B = \frac{6M_C}{t^2} = \frac{6 \times 498{\cdot}3}{36}$$
$$= 83{\cdot}05\ \text{N/mm}^2$$

The tensile load at this point will be

$$T_C = \frac{ph}{2} = \frac{0{\cdot}081 \times 200}{2}$$
$$= 8{\cdot}1\ \text{Newtons.}$$

Hence

$$\sigma_C = \frac{8{\cdot}1}{6} = 1{\cdot}35\ \text{N/mm}^2,$$

thus giving a combined tensile stress at C equal to

$\underline{84{\cdot}4\ \text{N/mm}^2}$.

(iv) Consider the cover itself. The horizontal stiffeners can be conveniently treated as simply supported beams under uniform pressure load. The bending moment at mid span is then given by

$$M_{mid} = \frac{pL^2}{8}$$

Here $p = 0{\cdot}069 \times 440 = 30{\cdot}36$ Newtons per unit length of span. Therefore

$$M_{mid} = 8538750\ \text{N.mm}$$

and the bending stress

$$\sigma_B = \frac{8538750}{74{\cdot}6 \times 1000}$$
$$= 114{\cdot}61\ \text{N/mm}^2$$

The direct tensile load

$$T_D = \frac{(0{\cdot}069 \times 440) \times 1500}{2}$$
$$= 22770\ \text{Newtons,}$$

which gives a direct tensile stress of

Fig. 18.

$\sigma_D = \frac{22770}{23{\cdot}4 \times 10}$
$= 9{\cdot}73$ N/

Hence the com
level =

$\sigma_T = \underline{124{\cdot}34}$ N

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out detailed ca
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Note that if
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the max. com
level would o
Compare this
136·50 N/mm
base of the ve
item (i).

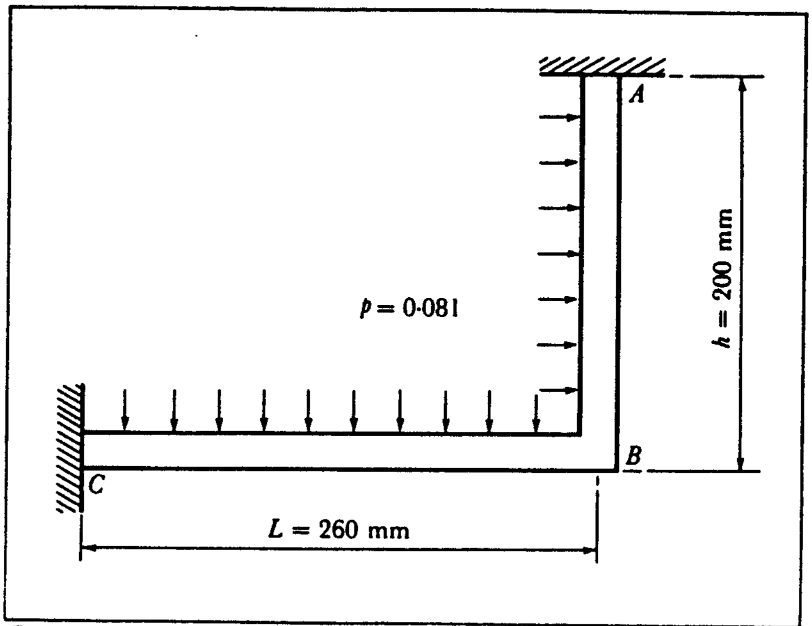


Fig. 18.

$$\sigma_D = \frac{22770}{23{\cdot}4 \times 100}$$
$$= 9{\cdot}73 \text{ N/mm}^2.$$

Hence the combined tensile stress level =

$$\sigma_T = \underline{124{\cdot}34 \text{ N/mm}^2.}$$

The above calculations indicate once again that the plate panels and the stiffener sections are adequate for the loading conditions specified.

Now that the overall design check has been made on the main components the designer can carry out detailed calculations on other features particular to his case, such as flanged joint details and the adequacy of supporting sections.

Note that if this enclosed tank had been of integral construction the max. combined tensile stress level would only be 101 N/mm^2 Compare this with the 136·50 N/mm^2 calculated at the base of the vertical stiffener under item (i).

3. Integral Rectangular Tanks – Ducting

(a) Uniform wall thickness throughout – no stiffeners

In such cases we simply determine, from Fig. 3, the max. bending moment and the direct tensile load at the corners, i.e.

$$M_B = \alpha_B p L^2 \quad \text{and} \quad T_B = \frac{pL}{2}$$

for a given h/L ratio.

The combined max. tensile stress is then given by

$$\sigma_T = \sigma_B + \sigma_D$$
$$= 6\alpha_B p\left(\frac{L}{t}\right)^2 + \frac{pL}{2t} \quad \ldots\ldots(18)$$

where L is the larger span and t is the plane thickness

By establishing the allowable design stress level for the plate material σ_{all}, either from the appropriate standard or other sources, we can then directly calculate the max. design pressure for that particular geometry (h/L, L/t ratios) by substituting σ_{all} for σ_T in Equation (18) and re-arranging to give the following relationship.

$$p = \frac{\sigma_{all}}{6 \times \alpha_B\left(\frac{L}{t}\right)^2 + \frac{L}{2t}} \quad \ldots\ldots\ldots(19)$$

Equation (19) has been used to establish the plots shown in Figs. 19 and 20 for an allowable design stress level of 155 N/mm^2 (22475 psi) for square and 2 : 1 ratio rectangular ducting, the latter proving to be the optimum design ratio. The band between the two curves (shown in Figure 19) covers the whole range of rectangular vessels from
$h/L = 0$ to $h/L = 1{\cdot}0$
or for
$L/h = 1{\cdot}0$ (i.e. square duct) to
$L/h =$ infinity which refers to built-in beams under uniform pressure load. These plots can also be used to determine the max. design pressure for different σ_{all} values or where the weld factor needs to be considered. In such instance the p value determined from Fig. 19 or 20 needs to be multiplied by

$$\frac{\sigma_{all}}{155} \quad \text{or} \quad \frac{\sigma_{all} \times E}{155}$$

as the case may be, where σ_{all} is the appropriate allowable design stress and E is the weld factor.

Fig. 20 also indicates how inefficient the rectangular cross-section vessels are in comparison with the cylindrical (circular cross-section) ones, the latter being able to carry 20 to 120 times higher pressures for equivalent size, for L/t range between 20 and 120 respectively.

(b) Uniform wall thickness with uniform section stiffeners

The approach is the same as for case (a) above with the exception that the uniform pressure p is replaced by $p \times$ pitch, i.e., ps in all the expressions containing the pressure term.

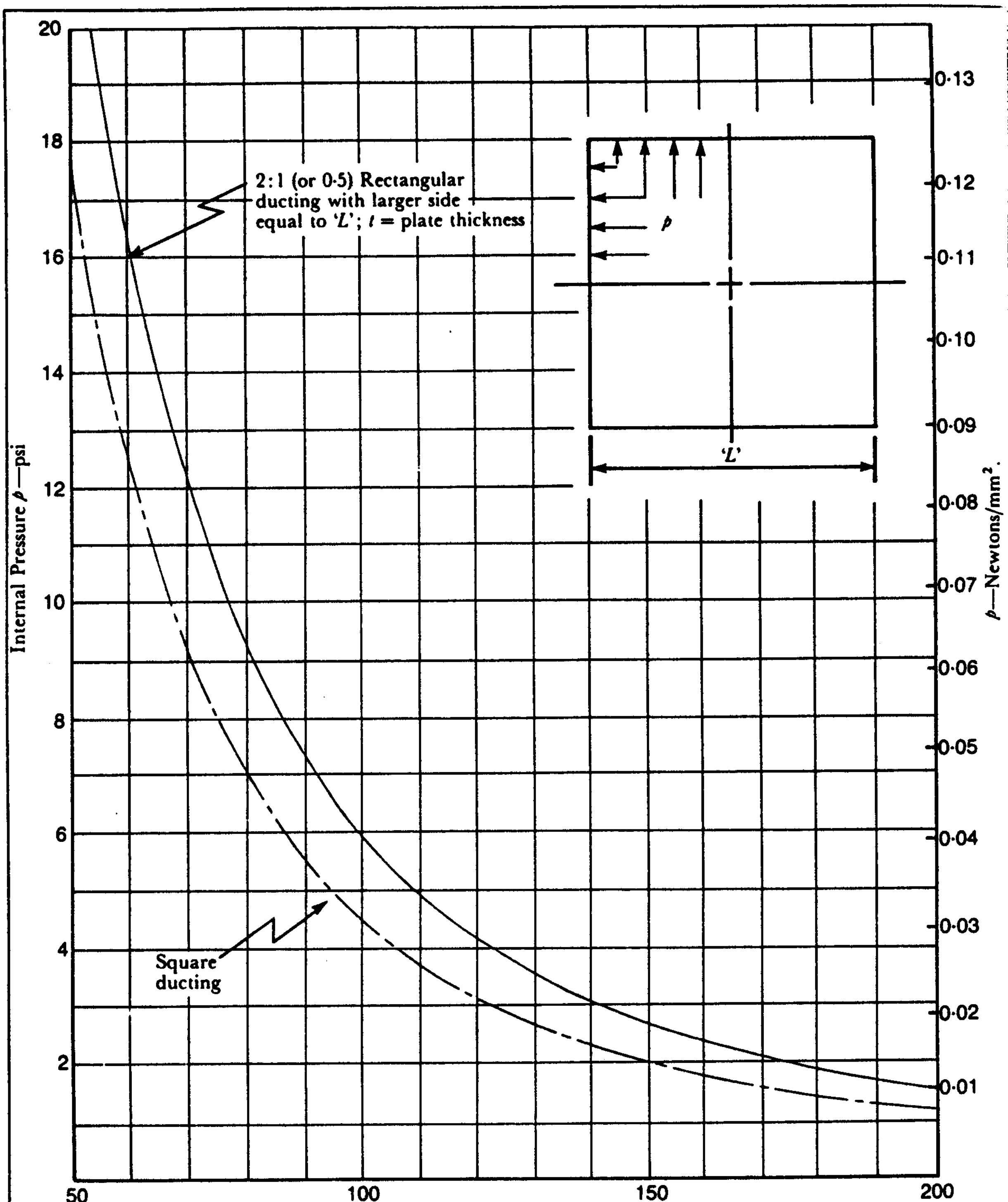

20
18
16
14
12
10
8
6
4
2
Internal Pressure p—psi
2:1 (or 0·5) Rectangular ducting with larger side equal to 'L'; t = plate thickness
p
'L'
Square ducting
0·13
0·12
0·11
0·10
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
p—Newtons/mm².
50
100
150
200
'L/t' Ratio

Fig. 19.

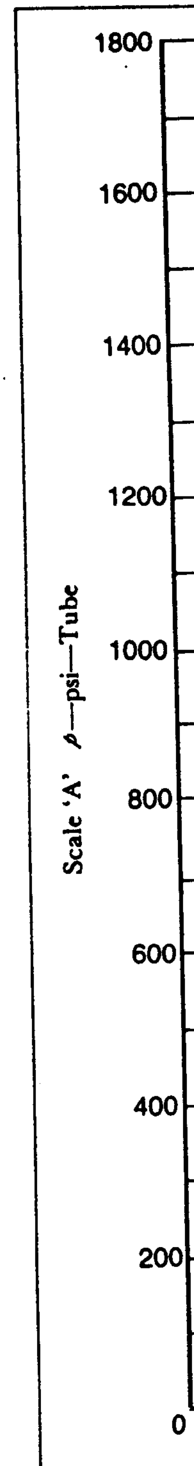
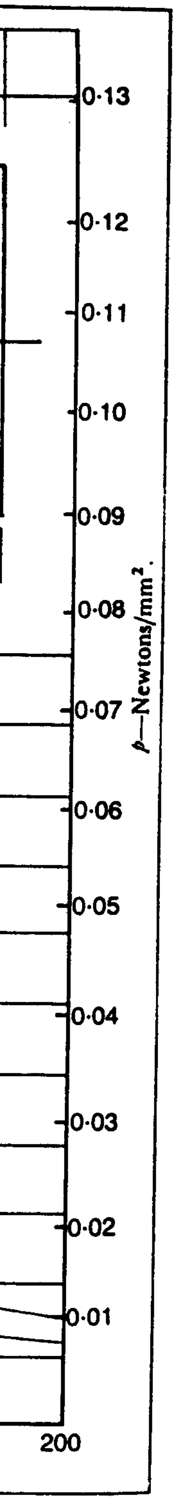

1800
1600
1400
1200
1000
800
600
400
200
0
Scale 'A' p—psi—Tube

Fig. 20.


0·13
0·12
0·11
0·10
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
200
p—Newtons/mm^2.

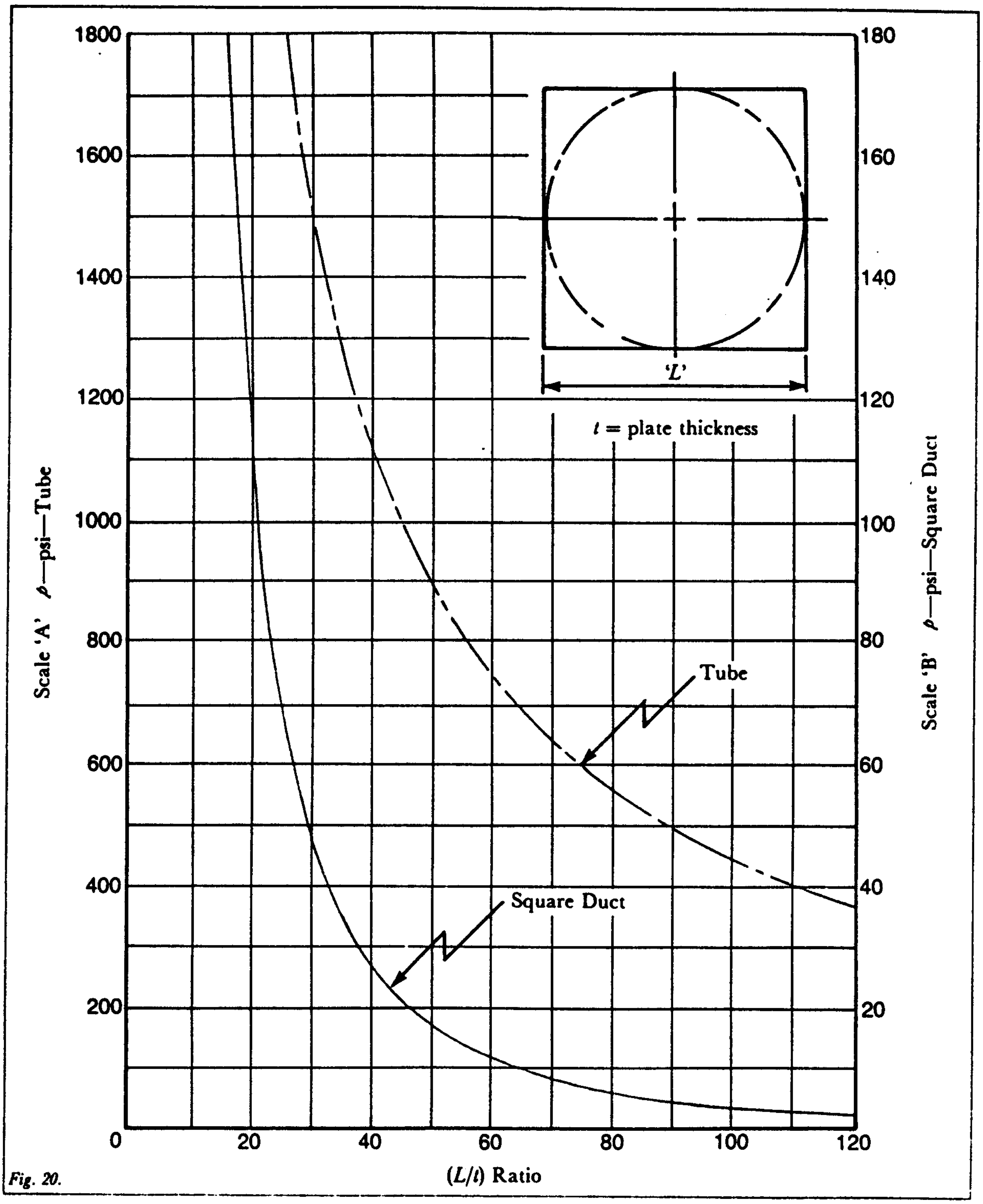

1800
1600
1400
1200
1000
800
600
400
200
0
180
160
140
120
100
80
60
40
20
20
40
60
80
100
120
'L'
t = plate thickness
Tube
Square Duct
Scale 'A' p—psi—Tube
Scale 'B' p—psi—Square Duct
(L/t) Ratio

Fig. 20.

In addition the plating between the stiffeners needs checking by making use of the information contained in Fig. 7.

Other Design Features

(1) Corner joint where the main stiffeners are not continuous, i.e. not welded at corner joints

In such instances the stiffener sections can be checked by treating these as simply supported beams of span L and h (as before) under the action of a uniform pressure load of $p \times s$, where s is the pitch or the distance between a pair of stiffeners. Hence the max. bending moments at mid span positions will be

$$M = 0{\cdot}0833(ps)L^2 \text{ for span } L,$$

and

$$M = 0{\cdot}0833(ps)h^2 \text{ for span } h.$$

The direct loads will be

$$T = \frac{psh}{2} \quad \text{and} \quad \frac{psL}{2}$$

respectively.

Having checked these we then need to assess the strength of the "corner angles" themselves.

Consider the construction detail shown below. (Such details do still occur on a number of hoppers or silos here as well as in other countries.)

The corner angle will be subjected to the direct loads

$$\frac{psL}{2} \quad \text{and} \quad \frac{psh}{2}$$

as well as a uniform pressure load p over the two arms of length l. These forces create a bending moment M as shown which cause the rotation of the two arms through an angle θ. This angle θ can be determined by the following method. For a simply supported beam of length L loaded by a uniform pressure of ps the ends will rotate through an angle θ_L given by

$$\theta_L = \frac{psL^3}{24EI_1} \quad \ldots\ldots\ldots\ldots\ldots (22)$$

where E = Youngs modulus, usually $29-30\times10^6$ psi, for common steels (207×10^3 N/mm^2), and I_1 is the moment of inertia of the beam section of span L.

Thus the combined angle of rotation

$$\theta = (\theta_L+\theta_h)\times\tfrac{1}{2}$$

$$= \frac{ps}{24E}\left(\frac{L^3}{I_1}+\frac{h^3}{I_2}\right)\times\frac{1}{2}$$

$$= \frac{ps}{48E}\left(\frac{L^3}{I_1}+\frac{h^3}{I_2}\right) \quad \ldots\ldots\ldots (23)$$

Now for a cantilever of length l, the end deflection due to an edge moment M_o is given by

$$\theta_o = \frac{M_o l}{EI_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Here

$$I_o = \frac{t^3 b}{12},$$

where t is the mean thickness of the angle arms.

Combining the two equations (23) and (24) we obtain $\theta = \theta_o$, so that

$$M_o = \frac{pst^3b}{576l}\left(\frac{L^3}{I_1}+\frac{h^3}{I_2}\right) \quad \ldots\ldots (25)$$

and the bending stress is given by

$$\sigma_B = \frac{6M_o}{bt^2}$$

$$= \frac{pst}{96l}\left(\frac{L^3}{I_1}+\frac{h^3}{I_2}\right) \quad \ldots\ldots\ldots (26)$$

The additional bending stress due to pressure load over the span l will normally be small by comparison with σ_B above and can be safely ignored.

Examination of Equation (26) indicates that the angle mean wall thickness will need to be fairly substantial in order to keep the bending stress levels within acceptable limits. It is much safer and more economic to weld the two main stiffeners together so that they form an integral rectangular structure.

(2) Floor or supporting structure – local buckling loads

As an example let us assume that we have a rectangular vessel which has a number of floor stiffeners of a certain cross-section.

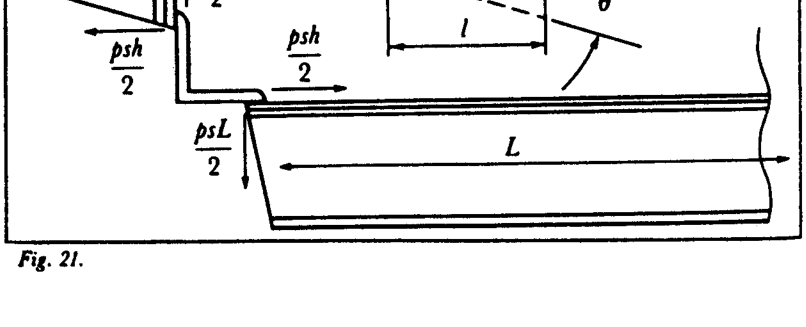


Fig. 21.

The vessel is rest
of discrete points
load per support

D = 257 mm o
d = 240 mm
b = 127 mm
t = 6·1 mm
$\frac{D}{t} = \frac{257}{6{\cdot}1}$ = 42
as shown in Fig.
Since D/t ratio f
section is over 3
angle of 60° for
that the effectiv
stiffener web tal
compressive loa
$b + 0{\cdot}866d$ = 1
= 3
Hence the cr
of the web actir
compression =
=
and the compre
$\sigma_C = \frac{10\times1000}{2043{\cdot}5}$
= 4·894 × 9
= 48·0 N/m
The length o
"column" = d
the ends of this
restrained in d
ends, the effec
$0{\cdot}7L$ = 0·7 × 2
(See BS449, A

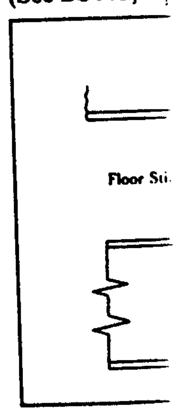


Fig. 22.

h^3 … I_2) ……(25)

ress is given by

) ………(26)

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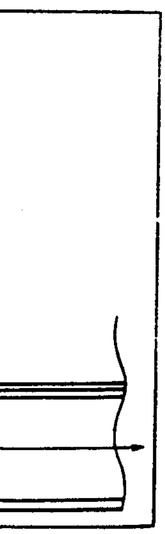

The vessel is resting on a number of discrete points so that the max. load per support = 10 tonnes.

D = 257 mm overall depth
d = 240 mm
b = 127 mm
t = 6·1 mm

$$\frac{D}{t} = \frac{257}{6 \cdot 1} = 42$$

as shown in Fig. 22.

Since D/t ratio for the stiffener section is over 35 suggest using the angle of 60° for θ shown below, so that the effective width of the floor stiffener web taking the compressive load of 10 tonnes =

$$b + 0{\cdot}866d = 127 + 208 = 335 \text{ mm}$$

Hence the cross-sectional area of the web acting as column under compression = 335 × 6·1 = 2043·5 mm²
and the compressive stress.

$$\sigma_C = \frac{10 \times 1000}{2043 \cdot 5} = 4{\cdot}894 \text{ kg/mm}^2$$

$$= 4{\cdot}894 \times 9{\cdot}81$$

$$= 48{\cdot}0 \text{ N/mm}^2$$

The length of the "column" = d = 240 mm. Since the ends of this column are restrained in direction at both ends, the effective length = $0{\cdot}7L = 0{\cdot}7 \times 240 = 168$ mm. (See BS449, Appendix D.)

The slenderness ratio for this column =

$$\frac{L}{r} = \frac{168}{1 \cdot 761} = 95{\cdot}4.$$

From Table 17a of BS 449, Part 2, 1969 the allowable compressive stress $p_c = 84{\cdot}5$ N/mm².

As $p_c > \sigma_c$ the floor stiffeners do not require reinforcing gussets.

Note that the radius of gyration r can be found from the following

A = cross-section area
= 2043·5 mm²
I = moment of inertia

$$= \frac{335 \times 6 \cdot 1^3}{12} = 6336{\cdot}553 \text{ mm}^4$$

But

$$I = Ar^2$$

so that

$$r^2 = \frac{I}{A}$$

$$= \frac{6336 \cdot 553}{2043 \cdot 5} = 3{\cdot}10$$

and hence

$$r = \sqrt{3 \cdot 10} = 1{\cdot}761 \text{ mm}$$

(3) Flanged Connections

Straightforward flanged connections are a common feature of low pressure ducting, hoppers, silos and storage tanks. Sometimes these are introduced in order to facilitate handling, transport and/or erection at site. On occasions the decision to introduce flanged connections is made following the release of the manufacturing drawing so that the designer is not really aware of their existence. On other occasions the designer or the manufacturer simply fail to appreciate the difference or the weakening effect of such an introduction on the strength of the component.

Consider a typical flange detail shown in Fig. 23.

The effective width of the flange resisting the bending moment can be taken as
$b = 2\tan 60° \times h = 3{\cdot}464h$. Note that if the two adjacent distances b so calculated overlap then $b = s$ the pitch between bolts.

Let F = the load per bolt pitch acting on the wall of the tank. For rectangular tank this force would be equal to

$\frac{psL}{2}$, see previous examples.

The moment at points B and C (see Case 9, Table I)

$$M = F \times h.$$

The section modulus of the vertical leg of the angle is given by

$$Z = \frac{bt^2}{6} = \frac{3{\cdot}464ht^2}{6}$$

and the bending stress then becomes

$$\sigma_B = \frac{M}{Z} = \frac{6Fh}{3{\cdot}464t^2} \quad \text{………(27)}$$

For a close pitched bolt the weakening effect of the bolt hole has to be considered. In this case the section modulus =

$$\frac{(s-d)\,t^2}{6},$$

where d = bolt hole diam. In calculating the bending moment $M = F \times h$ above we have ignored the effect of the pressure acting on BC as the flange opens. This effect

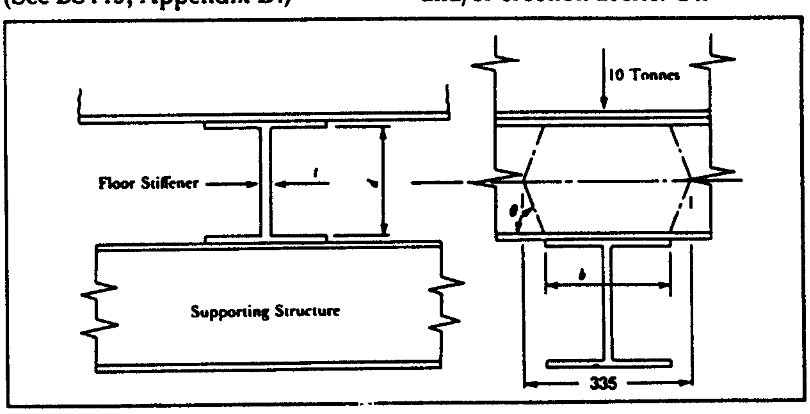

Fig. 22.

is negligible compared to the product Fh.

In addition to the above we need to check the stress level in the bolts.

The bolt load can be determined by taking the moment of forces about point A, the tip of the vertical leg of the flange, i.e.

$$F \times (AC) = L_B \times AB$$

where L_B = force in bolt. Therefore

$$L_B = \frac{F \times AC}{AB}$$

It is clear that depending on the two distances AC and AB the load experienced by each bolt can be considerably higher than F the nominal load per bolt pitch. Hence the distance h should be kept as small as possible (effectively making distance AB greater). This will also result in lower bending stress in the flange as given by Equation (27).

(4) Differential stiffener deflection

For the rectangular vessels of the type shown in Fig. 13 it may be worth while to check the central deflection of each stiffener by using the information contained in Fig. 5. This may be especially important for cases where the section properties (moment of inertia) vary for each stiffener. Where the central deflection for adjacent stiffeners differ considerably we need to check the additional bending moments imposed on the plate panels between the two stiffeners. This can be accomplished in the following manner. (Fig. 24.)

First calculate the central deflection for each stiffener using information given in Fig. 5. Then calculate the differential deflection between each adjacent pair of stiffeners. Let us call this differential deflection y. The additional bending moments imposed at the built-in edges of the plate panels are then given by

$$M_d = \frac{6EIy}{l^2},$$

where

E = Young's modulus

I = moment of inertia $= \frac{t^3}{12}$

l = distance between stiffeners

t = plate panel thickness

The above "correction" procedure is particularly important for plastic, or GRP tanks where the deflections can amount to several inches.

(5) Weld Factor (E)

Most of the codes tell us the weld factors we can use in the design calculation without specifying what this quantity really means. We know that the value of E (the weld factor) depends on what we do prior to, during and after fabrication, i.e., weld procedure, weld preparation detail, welding method and X-ray or ultrasonic examination. The more NDT employed the higher the weld factor.

One rational definition of the weld factor would be to imply that the quantity $(1-E)$ represents the size of the defect situated somewhere within the plate thickness t, as shown by an insert in Fig. 25.

The various equations given in the codes are (with minor correction factors) variations of the following:

Direct stress

$$\sigma_D = \frac{pD}{2tE} \quad \text{.................(20)}$$

and

$$\text{Bending stress } \sigma_B = \frac{\Upsilon p}{t^2 E} \quad \text{.....(21)}$$

or in the more usual form

$$t = \frac{pD}{2\sigma_D E} \quad \text{................(20a)}$$

$$t = \sqrt{\frac{\Upsilon p}{\sigma_B E}} \quad \text{...............(21a)}$$

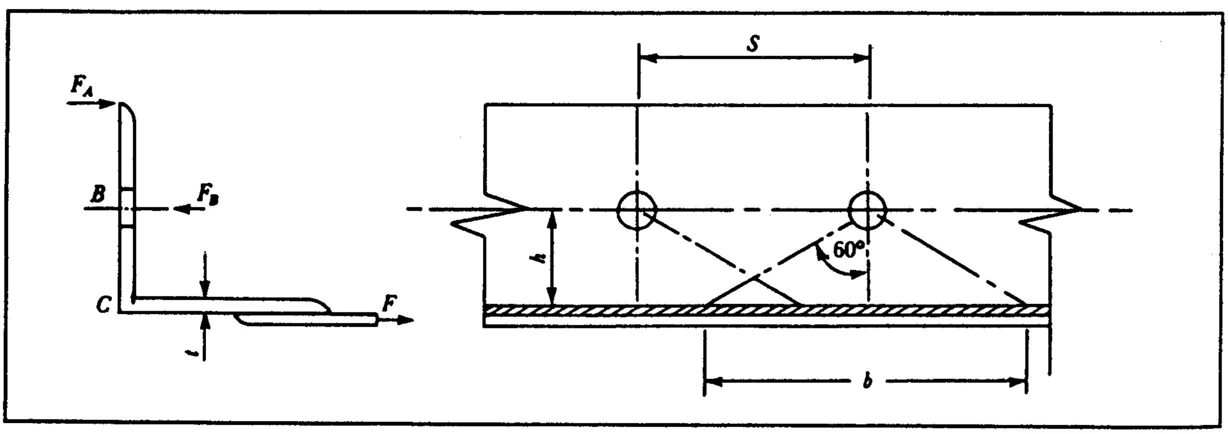

Fig. 23.

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$= \frac{\gamma p}{t^2 E}$ .....(21)

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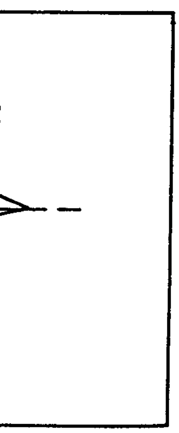

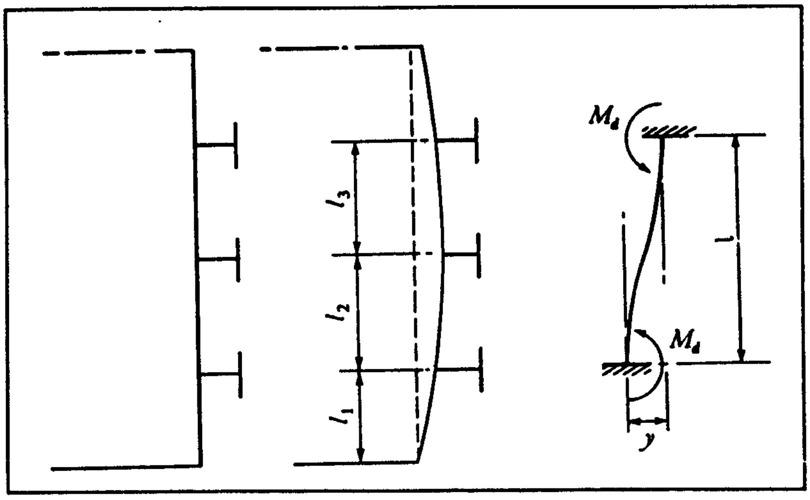

Fig. 24.

The above clearly indicates that the correction procedure is the same for the direct and bending stress conditions.

On closer examination this appears to be rather strange. To check the validity of this approach to predominately bending stress situations the writer has calculated the section modulus Z for a plate thickness t which contains a planar defect of size $(1-E)$ with the tip of this defect at a distance x from the free surface. The resultant plots for two weld factors, $E = 0{\cdot}75$ and $E = 0{\cdot}85$, are shown in Fig. 25.

The evidence shown here indicates that the present code

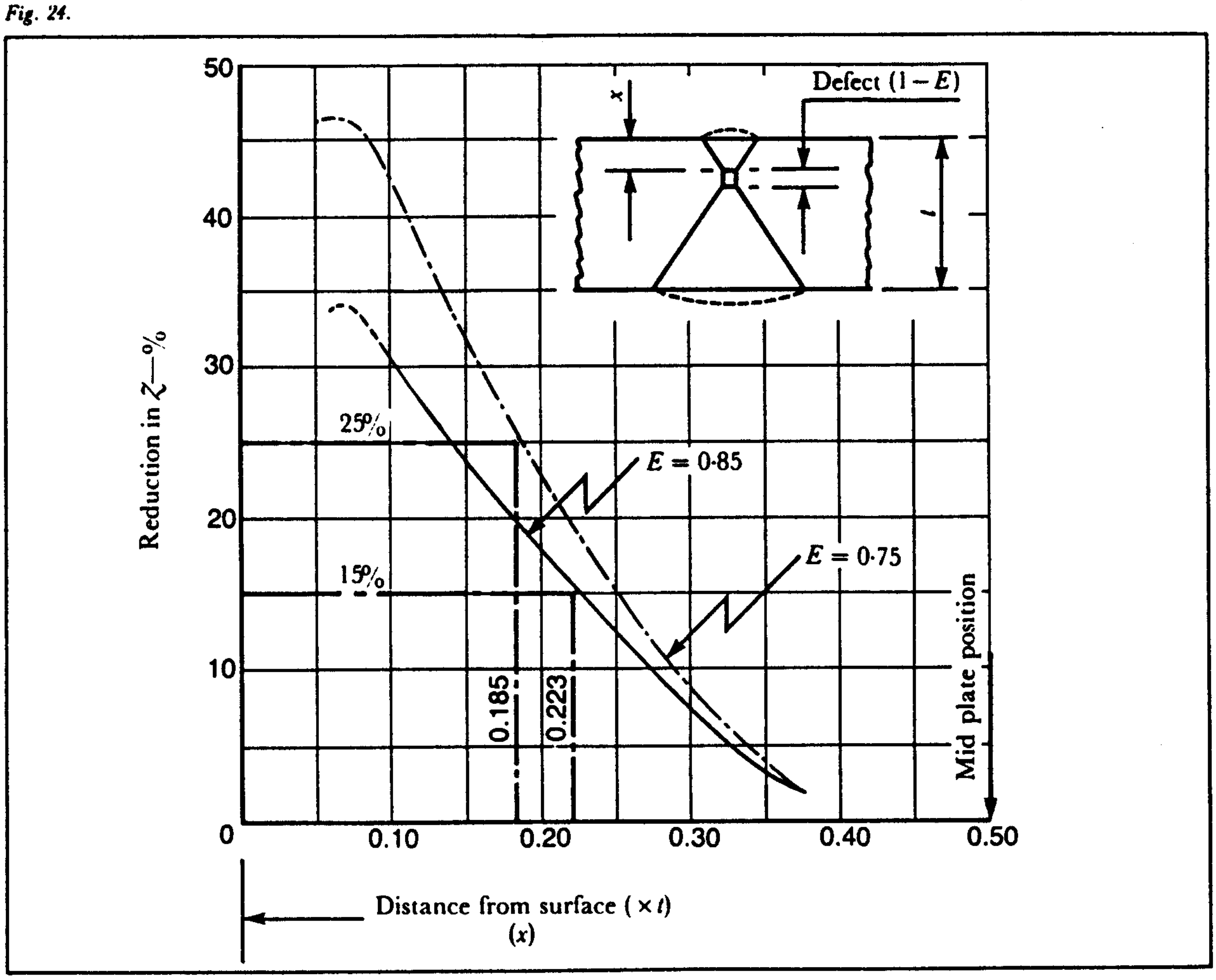

Fig. 25.

method of using the weld factor, as in Equations (20), (21), is safe for situations where the defects are entirely within the mid plate thickness, i.e. when x is within the limits of $0{\cdot}25t$ and $0{\cdot}75t$. The approach becomes increasingly unsafe when
(i) x becomes less than $0{\cdot}223t$ for $E = 0{\cdot}85$ case, and
(ii) x becomes less than $0{\cdot}185t$ for $E = 0{\cdot}75$ condition respectively.

One clear conclusion from the above is that for details subjected to predominately bending stresses the unfused land, lack of penetration, or lack of fusion, should occur within the central portion of the section. The local weld preparation should therefore be designed accordingly.

(6) Ligament Efficiency (E_L)—Rectangular Vessels

References 1, 3, 4 and 5 give various examples of how to calculate the ligament efficiency for various configuration of tube holes. All the efficiencies quoted seem to be based on the following standard equation

$$E_L = \frac{p - d_e}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

where p = pitch between uniformly spaced holes, d_e = mean effective tube hole diameter.

For plain holes the method of calculating the ligament efficiency is the same for both the direct, or membrane, and bending stress situations. The procedure is strictly applicable to flat plates which contain a series of holes. The stiffening effect of the nozzles themselves is completely ignored. Such vessels, with perforated side walls, probably do not exist. In practice the tube holes will be reinforced by the tube stubs as shown in Fig. 26. The height of the stub and reinforcement is given by

$$h = 0{\cdot}64\sqrt{d_n t},$$

where d_n is the mean nozzle dia.,

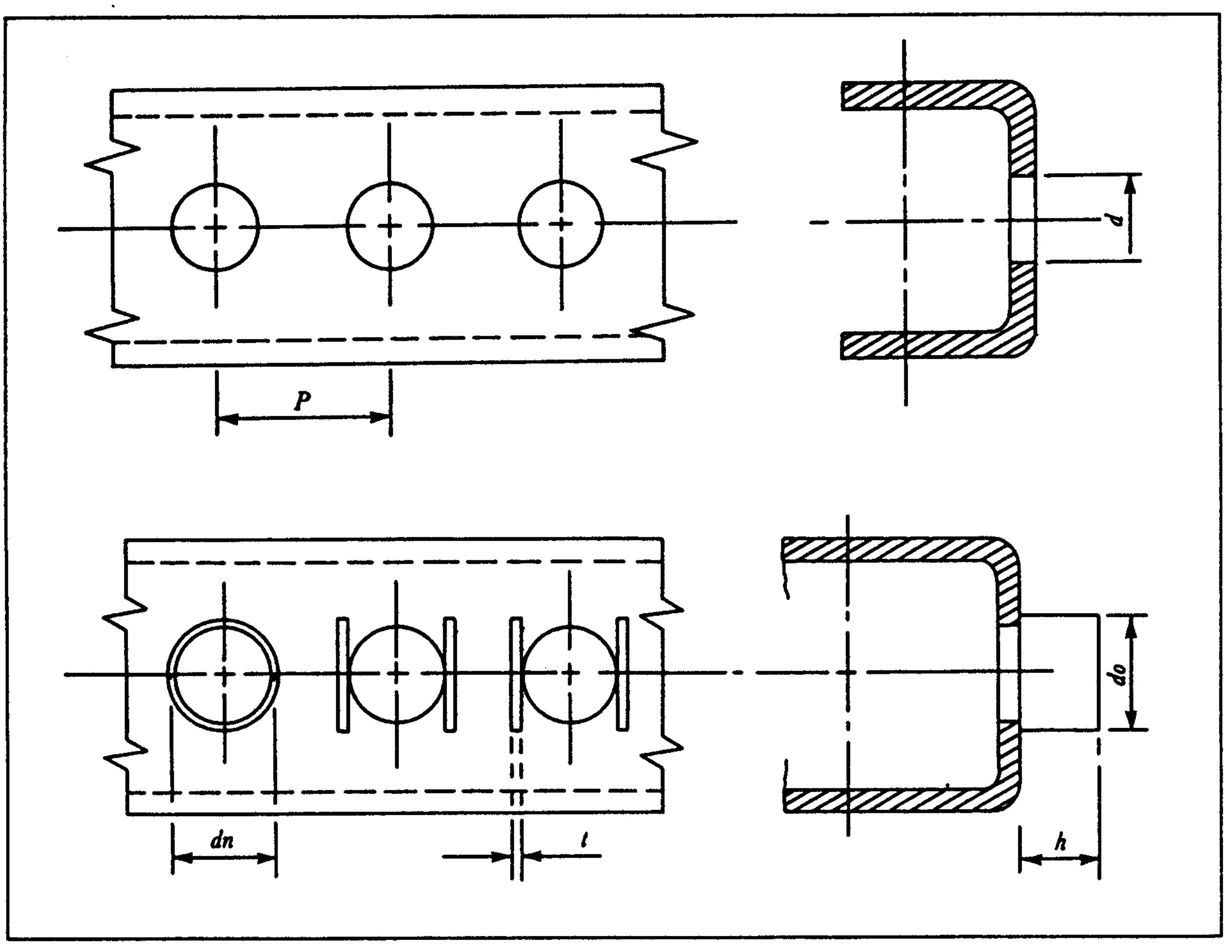

Fig. 26.

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Conclusions

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Appendix

The Basic equat
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and t is the nominal nozzle wall thickness.

From these two diagrams, the side containing the nozzle openings could in fact prove to be stronger than the plain (unperforated) wall since the former is effectively reinforced by a number of stiffeners. Thus the proposed method of calculating the ligament efficiency can be too conservative, especially for larger openings. Limited strain gauge proof tests carried out on similar heat exchangers seem to confirm this conservatism and the strengthening effects of nozzle stub ends on side wall panels.

Conclusions

The basic engineering theory outlined in this article shows how we can check the design of a number of non-circular cross-section pressure vessels.

The worked examples demonstrate how we can represent various details of a rectangular tank by replacing them with simplified geometries which can subsequently be evaluated by the fundamental engineering theory. We must endeavour to make each theoretical representation as close to the real component as possible. The closer the approximation between model and actual detail the higher the allowable design stress levels we can adopt.

The simplification procedure and the degree of sophistication needed will depend on how arduous will be the intended duty, on the confidence of our knowledge of the material properties and other factors. It is important to realise the implications of the simplifications and assumptions which have been made. If the theoretical model, or the simplified geometry, is too far detached from the real detail the design calculations may become invalid.

In other cases we can compensate for any gross simplifications by using much lower design stress levels or by ensuring that the theoretical representation is conservative. Personal experience and knowledge of the fundamental engineering theory will dictate the course of the appropriate action. This approach is certainly not recommended for the beginners. If you are one then seek advice.

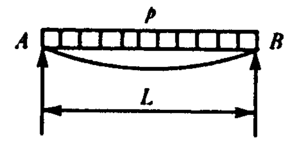
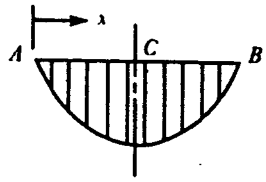
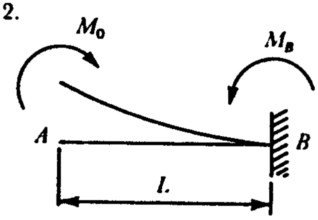
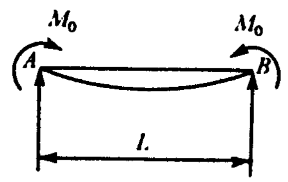
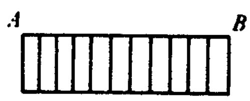
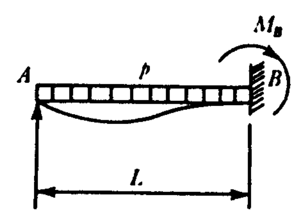
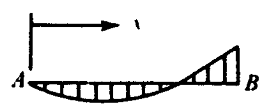
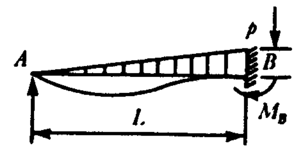
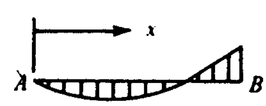
It is hoped that by outlining some of the salient features of the non-circular pressure vessels this article would prove useful to the designers and fabricators alike, and that it would, in some small way, lead to fewer failures of the type normally classified as due to poor or inadequate design.

References

1. ASME VIII, Division 1, 1980 Appendix 13.
2. The Theory and Practical Design of Bunkers by F. W. Lambert—BCSA.
3. The Swedish Pressure Vessels Code—1973.
4. British Standard 1113: 1969 Amendment Slip No. 1 issued 20.3.1972—Water Tube Boilers—Rectangular Section Headers.
5. Italian Standard ANCC—Section VSR1S: 1978 edition—Rectangular cross-section collector tanks.
6. British Standard 449: 1969—The use of Structural Steel in Building.
7. Formulas for Stress and Strain by Raymond J. Roark—McGraw Hill.
8. Rigid Frame Formulas by A. Kleinlogel—Crosby Lockwood.

Appendix

The Basic equations for moments, deflections and loads for the simple geometries considered in this article, are given overleaf in Table 1 for quick reference.

Loading system and geometry	Moment, Deflection, Rotation	Bending Moment Diagram
1. 	Max $M_C = \frac{pL^2}{8}$ $Y_C = \frac{5pL^4}{384EI}$ $\theta_{A,B} = \frac{pL^3}{24EI}$	 $Mx = \frac{1}{2}pL\left\{x - \frac{x^2}{L}\right\}$
2. 	$Y_A = \frac{M_0L^2}{2EI}$ $\theta_A = \frac{M_0L}{EI}$	
3. 	$Y_C = \frac{M_0L^2}{8EI}$ $\theta_{A,B} = \frac{M_0L}{2EI}$	
4. 	Max $M_B = \frac{pL^2}{8}$ $\theta_A = \frac{pL^3}{48EI}$	 $Mx = pL\left\{\frac{3}{8}x - \frac{x^2}{2L}\right\}$
5. 	Max $M_B = \frac{pL^2}{15}$ $\theta_A = \frac{pL^3}{60EI}$	 $Mx = \frac{1}{3}pL\left\{\frac{x}{5} - \frac{x^3}{3L^2}\right\}$

6. 
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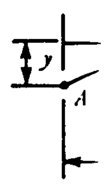
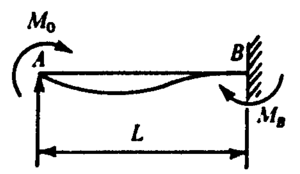
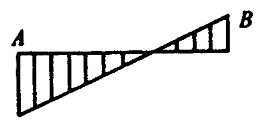
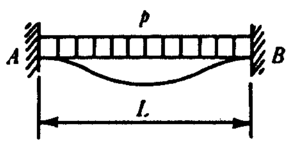
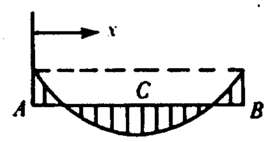
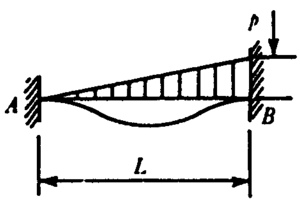
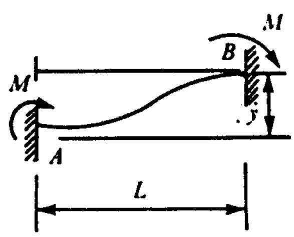
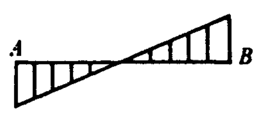
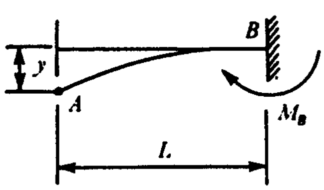
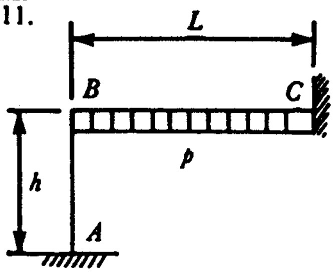
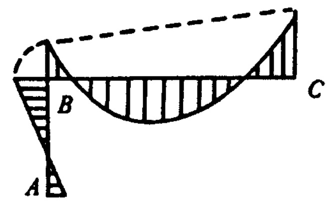
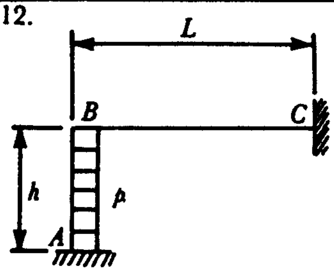
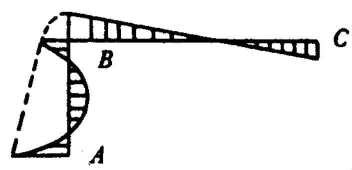
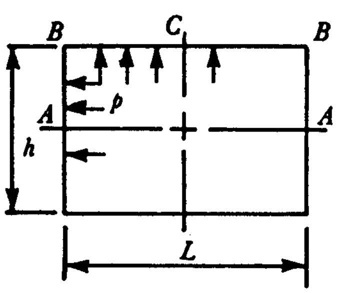
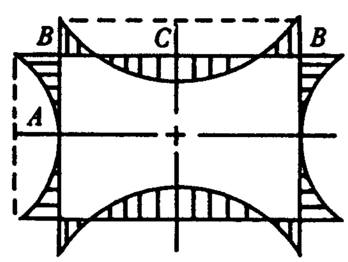
10. 

Loading system and geometry	Moment, Deflection, Rotation	Bending Moment Diagram
6. 	$M_B = \frac{M_0}{2}$ $\theta_A = \frac{M_0 L}{4EI}$	
7. 	Max $M_A = M_B = \frac{pL^2}{12}$ $M_C = \frac{pL^4}{24}$ $Y_C = \frac{pL^4}{384EI}$	 $M_x = \frac{1}{2}pL\left\{x - \frac{x^2}{L} - \frac{L}{6}\right\}$
8. 	Max $M_B = \frac{pL^2}{20}$ $y_{max} = \frac{pL^4}{764EI}$	 $M_x = \frac{1}{2}pL\left\{\frac{3x}{10} - \frac{L}{15} - \frac{x^3}{3L^2}\right\}$
9. 	$M = \frac{6EIy}{L^2}$	
10. 	$M_B = \frac{3EIy}{L^2}$ $\theta_A = \frac{3y}{L}$	

Loading system and geometry	Moment, Deflection, Rotation	Bending Moment Diagram
11. 	$M_A = \frac{pL^2}{24\mathcal{N}}$ $M_B = \frac{pL^2}{12\mathcal{N}}$ $T_{BA} = \frac{pL}{2}$ $M_C = \frac{pL^2(3k+2)}{24\mathcal{N}}$	
12. 	$M_A = -\frac{ph^2(2k+3)}{24\mathcal{N}}$ $M_B = -\frac{ph^2k}{12\mathcal{N}}$ $M_C = \frac{ph^2k}{24\mathcal{N}}$ $T_{BC} = \frac{ph}{2}$	
13. 	$M_A = \frac{pL^2}{8}\left\{\beta^2 - 1 + \frac{1}{3}\left(\frac{k+3-2\beta^2}{k+1}\right)\right\}$ $M_B = \frac{pL^2}{8}\left\{1 - \frac{1}{3}\left(\frac{k+3-2\beta^2}{k+1}\right)\right\}$ $M_C = \frac{pL^2}{24}\left\{\frac{k+3-2\beta^2}{k+1}\right\}$ $T_{BC} = \frac{ph}{2}$, $T_{BA} = \frac{pL}{2}$	

Note:
For cases Nos. 11, 12 and 13, moment of inertia I_1 refer to the longer spans L and I_2 refer to the shorter spans h respectively;

$$k = \frac{I_2}{I_1}\cdot\frac{h}{L}, \qquad \mathcal{N} = k + 1, \qquad \beta = \frac{h}{L}.$$

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